Subsystem of neutral mesons beyond the Lee–Oehme–Yang approximation

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Abstract

We begin with a discussion of the general properties of eigenvectors and eigenvalues for an effective Hamiltonian governing time evolution in a two state subspace of the state space of the total system under consideration. Next, the Lee, Oehme and Yang (LOY) theory of time evolution in such a subspace is considered. The CPT– and CP–symmetry properties of the LOY effective Hamiltonian are discussed. Next the CPT transformation properties of the exact effective Hamiltonian for two state subspace are discussed. Using the Khalfin Theorem we show that the diagonal matrix elements of the exact effective Hamiltonian governing the time evolution in the subspace of states of an unstable particle and its antiparticle need not be equal at for $t > t_0$ (t_0 is the instant of creation of the pair) when the total system under consideration is CPT invariant but CP noninvariant. (Suitable matrix elements of the LOY effective Hamiltonian are equal in such a case). The unusual consequence of this result is that, contrary to the properties of stable particles, the masses of the unstable particle "1" and its antiparticle "2" need not be equal for $t \gg t_0$ in the case of preserved CPT and violated CP symmetries. Also, basic assumptions necessary for the proof of the CPT Theorem are discussed. It is found that the CPT Theorem is not valid for a physical system with unstable particles decaying exponentially. From this property the conclusion is drawn that CPT–transformation cannot be a symmetry in a system which contains the LOY model as a subsystem, and, thus this model is shown to be incapable of describing possible CPT–violation effects correctly. Using an exact equation governing the time evolution in the subspace of the total state space we show that there exists an approximation which is more accurate than the LOY approximation, and which leads to an effective Hamiltonian whose diagonal matrix elements posses properties consistent with the conclusions obtained for the exact effective Hamiltonian. Using this more accurate approximation we show that the interpretation of the tests measuring the difference between the

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K_0 mass and the $\overline{K}_0$ mass as the CPT–symmetry test is wrong. We find that in fact such tests should rather be considered as tests for the existence of the hypothetical interaction allowing the first order $|\Delta S| = 2$ transitions $K_0 \rightleftharpoons \overline{K}_0$. We also discuss relations between some parameters describing properties of the neutral meson complex obtained within the LOY theory and those obtained using the more accurate approximation than the LOY one.

1 Introduction.

Many tests of fundamental symmetries consist in searching for decay processes of neutral mesons. A subsystem of neutral mesons forms a two particle complex in the total system under investigations. In the quantum decay theory of multiparticle complexes, properties of the transition amplitudes

$$A_{j,\psi}(t) = \langle u_j|\psi;t\rangle \tag{1}$$

and properties of the matrix elements of the effective Hamiltonian governing the time evolution in the subspace of states of the complex are usually analyzed. Here, vectors $\{|u_j\rangle\}_{j\in U} \in \mathcal{H}$, where $\mathcal{H}$ is the total state space of the system under consideration, represent the unstable states of the system considered, $\langle u_j|u_k\rangle = \delta_{jk}$, and $|\psi;t\rangle$ is the solution of the Schrödinger equation (we use $\hbar = c = 1$ units)

$$i\frac{\partial}{\partial t}|\psi;t\rangle = H|\psi;t\rangle, \tag{2}$$

having the following form

$$|\psi;t>= \sum_{j\in U} a_j(t)|u_j\rangle + \sum_J f_J(t)|\phi_J\rangle, \tag{3}$$

where vectors $|\phi_J\rangle \in \mathcal{H}$ describe the states of decay products, $\langle u_j|\phi_J\rangle = 0$ for every $j \in U$. The initial condition for Eq (2) in the case considered is usually assumed to be

$$\begin{aligned} |\psi;t=t_0\equiv 0\rangle &\stackrel{\text{def}}{=} |\psi\rangle \equiv \sum_{j\in U} a_j|u_j\rangle \in \mathcal{H}_{||}, \\ f_J(t=0) &= 0. \end{aligned} \tag{4}$$

where $\mathcal{H}_{||}$ is the subspace of $\mathcal{H}$ spanned by set of vectors $\{|u_j\rangle\}_{j\in U}$. In Eq (2) H denotes the complete (full), selfadjoint Hamiltonian of the system.

From (2) it follows that in the general case $|\psi;t\rangle = U(t)|\psi\rangle$, where

$$U(t) \equiv e^{-itH}, \tag{5}$$

is the total unitary evolution operator. So,

$$A_{j,\psi}(t) \equiv \langle u_j|U(t)|\psi\rangle. \tag{6}$$

It is not difficult to see that this property and hermiticity of H imply that

$$A_{j,j}(t)^* = A_{j,j}(-t). \tag{7}$$

Therefore, the decay probability of an unstable state (usually called the decay law), i.e., the probability for a quantum system to remain in its initial state $|\psi\rangle \equiv |u_j\rangle$

$$p_j(t) \stackrel{\text{def}}{=} |A_{j,j}(t)|^2 \equiv |a_j(t)|^2, \tag{8}$$

must be an even function of time:

$$p_j(t) = p_j(-t). \tag{9}$$

This last property suggests that in the case of the unstable states prepared at some instant t_0, say $t_0 = 0$, the initial condition (4) for the evolution equation (2) should be formulated more precisely. Namely, from (9) it follows that the probabilities of finding the system in the decaying state $|u_j\rangle$ at the instant, say $t = T \gg t_0 \equiv 0$, and at the instant $t = -T$ are the same. Of course, this can never occur. In almost all experiments in which the decay law of a given unstable particle is investigated this particle is created at some instant of time, say t_0, and this instant of time is usually considered as the initial instant for the problem. From the property (9) it follows that the instantaneous creation of the unstable particle is impossible. For the observer, the creation of this particle (i.e., the preparation of the state, $|u_j\rangle$, representing the decaying particle) is practically instantaneous. What is more, using suitable detectors he is usually able to prove that it did not exist at times $t < t_0$. Therefore, if one looks for the solutions of the Schrödinger equation (2) describing properties of the unstable states prepared at some initial instant t_0 in the system, and if one requires these solutions to reflect situations described above, one should complete initial conditions (4) for Eq (2) by assuming additionally that

$$a_j(t < t_0) = 0, \quad (j \in U), \tag{10}$$

and that, for the problem, time t varies from $t = t_0 > -\infty$ to $t = +\infty$ only.

Amplitudes of type $a_j(t)$ can be calculated directly by solving the evolution equation (2), or by using the Schrödinger–like evolution equation governing the time evolution in $\mathcal{H}_{||}$. Searching for the properties of two particle subsystems one usually uses the following equation of the type mentioned [1] — [22] instead of Eq (2),

$$i\frac{\partial}{\partial t}|\psi;t\rangle_{\|} = H_{\|}|\psi;t\rangle_{\|}, \tag{11}$$

where $|\psi;t\rangle_{||} \in \mathcal{H}_{||}$ and by $H_{\|}$ we denote the effective nonhermitian Hamiltonian, which in general can depend on time t [23],

$$H_{\|} \equiv M - \frac{i}{2}\Gamma, \tag{12}$$

and

$$M = M^{+}, \;\; \Gamma = \Gamma^{+}, \tag{13}$$

are (2×2) matrices, acting in a two–dimensional subspace $\mathcal{H}_{\|}$ of the total state space $\mathcal{H}$. M is called the mass matrix, Γ is the decay matrix [1] — [22]. In many papers it is assumed that the real parts, $\Re(.)$, of the diagonal matrix elements of $H_{\|}$:

$$\Re\,(h_{jj}) \equiv M_{jj} \stackrel{\rm def}{=} M_{j}, \;\; (j = 1, 2), \tag{14}$$

where

$$h_{jk} = < \mathbf{j}|H_{\|}|\mathbf{k} >, \; (j, k = 1, 2), \tag{15}$$

correspond to the masses, M_1, M_2, of particle "1" and its antiparticle "2" respectively [1] — [22], (and such an interpretation of $\Re\,(h_{11})$ and $\Re\,(h_{22})$ will be used in this paper), whereas the imaginary parts, $\Im(.)$,

$$-\,2\Im\,(h_{jj}) \equiv \Gamma_{jj} \stackrel{\rm def}{=} \Gamma_{j}, \;\; (j = 1, 2), \tag{16}$$

are interpreted as the decay widths of these particles [1] — [22].

The standard method of derivation of such a $H_{\|}$ is based on a modification of the Weisskopf–Wigner (WW) approximation [24]. Lee, Oehme and Yang (LOY) adapted the WW approach to the case of a two particle subsystem [1] — [6] to obtain their effective Hamiltonian $H_{\|} \equiv H_{LOY}$. Almost all properties of the neutral kaon complex, or another two state subsystem, can be described by solving Eq (11) [1] — [22], with the initial condition corresponding to (4) and (10)

$$\begin{aligned} |\psi; t = t_0\rangle_{\|} \;\; &\equiv \;\; |\psi\rangle_{\|}, \\ \| \;|\psi; t = t_0\rangle_{\|}\| \;\; &= \;\; 1, \;\; |\psi; t < t_0\rangle_{\|} = 0, \end{aligned} \tag{17}$$

for $|\psi; t\rangle_{\|}$ belonging to the subspace $\mathcal{H}_{\|} \subset \mathcal{H}$ spanned, e.g., by orthonormal neutral kaons states $|K_0\rangle$, $|\overline{K}_0\rangle$, and so on, (then states corresponding to the decay products belong to $\mathcal{H} \ominus \mathcal{H}_{\|} \stackrel{\rm def}{=} \mathcal{H}_{\perp}$),

$$|\psi\rangle_{\|} \equiv a_1|\mathbf{1}\rangle + a_2|\mathbf{2}\rangle, \tag{18}$$

and $|\mathbf{1}\rangle$ stands for the vectors of the $|K_0\rangle$, $|B_0\rangle$, etc., type and $|\mathbf{2}\rangle$ denotes states of $|\overline{K}_0\rangle$, $\overline{B}_0\rangle$ type, $\langle \mathbf{j}|\mathbf{k}\rangle = \delta_{jk}$, $j, k = 1, 2$.

Defining projectors

$$P = |\mathbf{1}\rangle\langle\mathbf{1}| + |\mathbf{2}\rangle\langle\mathbf{2}|, \quad Q = I - P, \tag{19}$$

one has

$$\mathcal{H}_{||} = P\mathcal{H} \ni P|\psi; t\rangle \stackrel{\rm def}{=} |\psi; t\rangle_{||}, \tag{20}$$

and

$$\mathcal{H}_{\perp} = Q\mathcal{H} \ni Q|\psi; t\rangle \stackrel{\rm def}{=} |\psi; t\rangle_{\perp}. \tag{21}$$

Solutions of Eq. (11) can be written in a matrix form and such a matrix defines the evolution operator (which is usually nonunitary) $U_{\|}(t)$ acting in $\mathcal{H}_{\|}$:

$$|\psi;t\rangle_{\|} = U_{\|}(t)|\psi;t_0 = 0\rangle_{\|} \stackrel{\text{def}}{=} U_{\|}(t)|\psi\rangle_{\|}. \tag{22}$$

The problem of testing such fundamental symmetries like parity, CP–symmetry, or CPT–invariance, experimentally has attracted the attention of physicists, practically since the discovery of antiparticles. An especially important problem seems to be the problem of verifying if CPT symmetry is the symmetry of the Nature. There is a known theorem (called the CPT Theorem) in axiomatic quantum field theory that CPT–invariance must hold. This theorem is based on very general assumptions [25] — [31]: it requires for its proof that a quantum field theory be constructed from fields belonging to finite–dimensional representations of the Lorentz group, have a local interactions invariant under the proper Lorentz group, and the spectral condition be fulfilled (all energies must be nonnegative). CPT–invariance is the minimal condition for the existence of antiparticles within quantum field theory. Many tests of CPT–invariance consist in searching for decay processes of neutral kaons. It was realized almost from the discovery of CP violation [32] that it was important to study in detail the effective Hamiltonian, $H_{||}$, which describes the time evolution in the $K_0, \overline{K}_0$ complex [1] — [22]. In the large literature, all CP– and CPT–violation parameters in this complex are expressed in terms of matrix elements of this Hamiltonian. The old and new [5] — [7], [15, 16],[17] — [22] experimental tests of such fundamental symmetries as the CP–nonivariance and of the CPT–invariance in the neutral mesons complex need a correct interpretation of the parameters measured, which is independent of the approximations used for the theoretical description of the behavior of such a complex. The aim of this paper is to confront the standard interpretation of these parameters with the general, basic principles of quantum theory.

The paper is organized as follows: We begin from the discussion of Lee, Oehme and Yang theory, then we confront CP– and CPT–transformation properties of the LOY effective Hamiltonian with corresponding properties of the exact effective Hamiltonian for two particle complex under consideration. We also confront assumptions of the Lee, Oehme and Yang theory with assumptions necessary for the proof of the CPT Theorem. Also, an alternative approximation, (different from that used by Lee, Oehme and Yang), leading to the effective Hamiltonian, which CP– and CPT–transformation properties are consistent with such the properties of the Hamiltonian of the total system considered, is presented.

2 Preliminaries

2.1 The eigenvalue problem in two–dimensional subspace.

In the general case, the eigenvectors for $H_{||}$ are identified with quasistationary states in the subspace of states, $\mathcal{H}_{\|}$, of neutral meson complex under studies. When studying the eigenvalue problem for the (2×2) matrices, say $\mathbf{E}$, acting in $\mathcal{H}_{\|}$, one must solve the equation

$$\det |\mathbf{E} - \zeta I_{\|}| = 0. \tag{23}$$

Here $I_{\|}$ is the unity in $\mathcal{H}_{\|}$ - of course, $I_{\|} \equiv P$. Sometimes it is useful to rewrite the matrix $\mathbf{E}$ in terms of the Pauli matrices. In such a case we have

$$\mathbf{E} = E_0 I_{||} + \vec{E} \bullet \vec{\sigma}, \tag{24}$$

where

$$\begin{aligned}
\vec{E} \bullet \vec{\sigma} &= E_x\sigma_x + E_y\sigma_y + E_z\sigma_z, && (25)\\
E_0 &= \frac{1}{2}(E_{11} + E_{22}), && (26)\\
E_z &= \frac{1}{2}(E_{11} - E_{22}), && (27)\\
E^2 &= \vec{E} \bullet \vec{E} = E_x^2 + E_y^2 + E_z^2 && (28)\\
&\equiv E_{12}\, E_{21} + E_z^2,
\end{aligned}$$

where σ_ν, $(\nu = x, y, z)$, are the Pauli matrices.

Solutions of Eq. (23), i.e., the eigenvalues ζ for $\mathbf{E}$ can be expressed in terms of matrix elements E_{jk} of $\mathbf{E}$:

$$\begin{aligned}
\zeta_\pm &= \frac{1}{2}\Big(E_{11} + E_{22} \pm \{[E_{11} - E_{22}]^2 \ + \ 4E_{12}E_{21}\}^{1/2}\Big) && (29)\\
&\equiv E_0 \pm E, && (30)
\end{aligned}$$

The eigenstates of $\mathbf{E}$ are linear combinations of time–independent vectors $|\mathbf{1}\rangle$ and $|\mathbf{2}\rangle$:

$$|e_\pm\rangle = p_\pm|\mathbf{1}\rangle + q_\pm|\mathbf{2}\rangle. \tag{31}$$

The coefficients of such combinations are determined by the solution to the matrix equation:

$$[\mathbf{E} - \zeta_\pm I_{\|}] \begin{pmatrix} p_\pm \\ q_\pm \end{pmatrix} = 0, \tag{32}$$

from which one obtains

$$\frac{q_\pm}{p_\pm} = -\frac{E_{11} - \zeta_\pm}{E_{12}} \equiv -\frac{E_{21}}{E_{22} - \zeta_\pm}. \tag{33}$$

There are two, in general, nonorthogonal eigenvectors for $\mathbf{E}$:

$$|e_{\pm}\rangle \equiv p_{\pm}(\,|\mathbf{1}\rangle - \alpha_{\pm}|\mathbf{2}\rangle), \tag{34}$$

where

$$\begin{aligned} \alpha_{\pm} = -\frac{q_{\pm}}{p_{\pm}} &\equiv \frac{E_{11} - \zeta_{\pm}}{E_{12}} \\ &\equiv \frac{E_z \mp E}{E_{12}}. \end{aligned} \tag{35}$$

Requiring that $|e_{\pm}\rangle$ be normalized, one obtains

$$|p_{\pm}|^2 = \frac{1}{1 + |\alpha_{\pm}|^2}.$$

Eigenvectors and eigenvalues for $H_{\|}$ and $U_{\|}$ can be obtained inserting into these formulae matrix elements of $H_{\|}$ and $U_{\|}$ instead of $\mathbf{E}$.

Note that in the general case neither eigenvectors for $H_{||}$ nor for $U_{||}$ are orthogonal due to the nonhermicity of $H_{||}$ and nonunitarity of $U_{||}$.

2.2 Definitions and properties of C, P and T transformations.

Now let us discuss some consequences of the assumed CP– or CPT–invariance of the system under considerations, i.e., implications of the following assumptions

$$[\mathcal{CP}, H] = [\mathcal{CP}, P] = 0, \tag{36}$$

and

$$[\Theta, H] = 0, \tag{37}$$

$$[\Theta, P] = 0, \tag{38}$$

where $\Theta = \mathcal{CPT}$ and $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$ denote operators realizing charge conjugation, parity and time reversal respectively, for vectors in $\mathcal{H}$.

Under transformations: unitary $\mathcal{P}$, $\mathcal{C}$, $\mathcal{CP}$, and antiunitary: $\mathcal{T}$, Θ, the vectors, say $|\Psi_k; \mathbf{p_k}, \lambda_k\rangle \in \mathcal{H}$, where $\mathbf{p}_k$ and λ_k denote momentum and spin of particles Ψ_k respectively, behave as follows [33, 34]

$$\begin{aligned} \mathcal{C}|\Psi_k; \mathbf{p}_k, \lambda_k\rangle &= \beta_k^C|\overline{\Psi}_k; \mathbf{p}_k, \lambda_k\rangle, \\ \mathcal{P}|\Psi_k; \mathbf{p}_k, \lambda_k\rangle &= \beta_k^P|\Psi_k; -\mathbf{p}_k, -\lambda_k\rangle, \\ \mathcal{CP}|\Psi_k; \mathbf{p}_k, \lambda_k\rangle &= \beta_k^{CP}|\overline{\Psi}_k; -\mathbf{p}_k, -\lambda_k\rangle, \qquad (39) \\ \langle\mathcal{T}\Psi_j|\mathcal{T}\Psi_k\rangle &= \langle\Psi_k|\Psi_j\rangle, \\ \langle\Theta\Psi_j|\Theta\Psi_k\rangle &\equiv \langle\overline{\Psi}_j|\overline{\Psi}_k\rangle = \langle\Psi_k|\Psi_j\rangle, \qquad (40) \end{aligned}$$

where: $|\beta_k^C| = |\beta_k^{CP}| = 1$ and $\overline{\Psi}_k$ denotes an antiparticle for Ψ_k. There is $\mathcal{CP} = (\mathcal{CP})^+ = (\mathcal{CP})^{-1}$ and therefore eigenvalues for $\mathcal{CP}$ are equal ± 1. The parity of two-particle state $|\Psi_1, \Psi_2; \mathbf{p}_1, \lambda_1; \mathbf{p}_2, \mathbf{p}_2; L\rangle$, where L is a relative angular momentum, equals

$$\mathcal{P}|\Psi_1, \Psi_2; \mathbf{p}_1, \lambda_1; \mathbf{p}_2, \mathbf{p}_2; L\rangle = \beta_1^P \beta_2^P (-1)^L |\Psi_1, \Psi_2; -\mathbf{p}_1, -\lambda_1; -\mathbf{p}_2, -\lambda_2; L\rangle. \quad (41)$$

The parity of one pion state is $\beta_\pi^P = -1$ and therefore properties (39), (41) imply that there is $\beta_\pi^P \beta_\pi^P (-1)^L \stackrel{\text{def}}{=} \beta_{\pi,\pi}^P \equiv +1$ for the two-pion state with a zero relative angular momentum L. This means that [33]

$$\mathcal{CP}|\pi, \pi; L = 0\rangle = (+1)|\pi, \pi; L = 0\rangle. \quad (42)$$

Let L be the relative momentum of a two-pion subsystem and let L' be the angular momentum of third pion about the center of mass of the two-pion subsystem then [33]

$$\mathcal{CP}|\pi, \pi, \pi; L = 0, L' = 0\rangle = (-1)^{1+L'=0}|\pi, \pi, \pi; L = 0, L' = 0\rangle. \quad (43)$$

The following phase convention for neutral kaons is commonly used

$$\mathcal{CP}|\mathbf{1}\rangle = (-1)|\mathbf{2}\rangle, \qquad \mathcal{CP}|\mathbf{2}\rangle = (-1)|\mathbf{1}\rangle, \quad (44)$$

and

$$\Theta|\mathbf{1}\rangle = e^{-i\theta}|\mathbf{2}\rangle, \qquad \Theta|\mathbf{2}\rangle = e^{-i\theta}|\mathbf{1}\rangle. \quad (45)$$

Note that vectors

$$|K_{1(2)}\rangle \stackrel{\text{def}}{=} \frac{1}{\sqrt{2}}(\,|\mathbf{1}\rangle - (+)|\mathbf{2}\rangle), \quad (46)$$

are normalized, orthogonal

$$\langle K_j|K_k\rangle = \delta_{jk}, \quad (j, k = 1, 2), \quad (47)$$

eigenvectors of $\mathcal{CP}$ transformation,

$$\mathcal{CP}|K_{1(2)}\rangle = +(-1)|K_{1(2)}\rangle, \quad (48)$$

for the eigenvalues $+1$ and -1 respectively.

Now let us analyze some general properties of the neutral kaon complex caused by CP symmetry and independent of the approximation used to describe time evolution in neutral kaon complex. The conservation of CP implies that the decay modes, and hence the lifetimes of K_1 and K_2 must be different. The mass of K_0 is $m_K \simeq 498$ MeV, the masses of pions are $m_{\pi^\pm} \simeq 140$ MeV and $m_{\pi^0} \simeq 135$ MeV, so two and three pions decays are energetically allowed for neutral kaons. If $\mathcal{CP}$ symmetry is conserved then only allowed decay are $K_1 \longrightarrow 2\pi$, $K_2 \longrightarrow 3\pi$. K_2 cannot decay

in two pions at all. These conclusions can be easily derived from properties of decay amplitudes $A(K \to 2\pi)$, $A(\overline{K} \to 2\pi)$, etc.. Defining

$$A(K \to 2\pi) \stackrel{\text{def}}{=} \langle \pi\pi | H | \mathbf{1} \rangle, \quad A(\overline{K} \to 2\pi) \stackrel{\text{def}}{=} \langle \pi\pi | H | \mathbf{2} \rangle, \tag{49}$$

and so on, and using relations (42), (44) one finds in the case of conserved CP–symmetry

$$\begin{aligned} A(K \to 2\pi) &= \langle \pi\pi | H | \mathbf{1} \rangle \qquad (50) \\ &\equiv \langle \pi\pi | (\mathcal{CP})^{-1} (\mathcal{CP}) H (\mathcal{CP})^{-1} (\mathcal{CP}) | \mathbf{1} \rangle \\ &= \langle \mathcal{CP}(\pi\pi) | [(\mathcal{CP})(\mathcal{CP})^{-1}] H (\mathcal{CP}) | \mathbf{1} \rangle) \\ &= -\langle \pi\pi | H | \mathbf{2} \rangle \equiv -A(\overline{K} \to 2\pi). \qquad (51) \end{aligned}$$

Analogous considerations lead to the property

$$A(K \to 3\pi) = A(\overline{K} \to 3\pi). \tag{52}$$

Using these relations, the decay amplitudes $K_{1(2)} \longrightarrow 2\pi$ can be derived as

$$\begin{aligned} A(K_1 \to 2\pi) &\equiv \langle \pi\pi | H | K_1 \rangle \qquad (53) \\ &\equiv 2^{-1/2} [A(K \to 2\pi) - A(\overline{K} \to 2\pi)] \\ &= 2^{1/2} A(K \to 2\pi), \qquad (54) \end{aligned}$$

$$\begin{aligned} A(K_2 \to 2\pi) &\equiv \langle \pi\pi | H | K_1 \rangle \qquad (55) \\ &\equiv 2^{-1/2} [A(K \to 2\pi) + A(\overline{K} \to 2\pi)] \equiv 0. \qquad (56) \end{aligned}$$

For three pion decays mode one obtains in a similar manner

$$\begin{aligned} A(K_1 \to 3\pi) &\equiv \langle \pi\pi\pi | H | K_1 \rangle \equiv 0, \qquad (57) \\ A(K_2 \to 3\pi) &\neq 0. \end{aligned}$$

Thus we see that certain decay modes which are allowed for K_1 are forbidden for K_2 and vice versa. This is the essence of Gell– Mann ad Pais theory [33, 34, 35]. The difference in the allowed transitions implies a corresponding difference in the lifetimes τ. It should be $\tau_{K_1} \ll \tau_{K_2}$ because $\tau_{K_2} \sim (m_K - 3m_\pi)^{-1}$ and $\tau_{K_1} \sim (m_K - 2m_\pi)^{-1}$, i.e., the decay rate for two pion decay is larger then for three pion decays.

2.3 Properties of eigenvectors for $H_{||}$.

Generally, in the case of two dimensional subspace $\mathcal{H}_{||}$ the eigenvectors of $H_{\|}$ acting in this $\mathcal{H}_{||}$ will be denoted as $|l\rangle, |s\rangle$. The solutions of the eigenvalue problem (23)

can be easily adopted to this case by identifying eigenvectors $|e_\pm\rangle$ of $\mathbf{E}$ with $|l(s)\rangle$,

$$
\begin{aligned}
|e_{+(-)}\rangle \;\rightarrow |l(s)\rangle &= \frac{1}{\sqrt{|p_{l(s)}|^2+|q_{l(s)}|^2}}\Big(p_{l(s)}|\mathbf{1}\rangle + q_{l(s)}|\mathbf{2}\rangle\Big) && (58)\\
&\equiv \rho_{l(s)}\Big(|\mathbf{1}\rangle - \alpha_{l(s)}|\mathbf{2}\rangle\Big), && (59)
\end{aligned}
$$

$$
\frac{q_{l(s)}}{p_{l(s)}} = -\frac{h_{11}-\mu_{l(s)}}{h_{12}} \equiv -\frac{h_{21}}{h_{22}-\mu_{l(s)}}, \tag{60}
$$

$$
\alpha_{l(s)} = \frac{h_z - (+)h}{h_{12}}, \tag{61}
$$

and replacing eigenvalues $\zeta_\pm$ of $\mathbf{E}$ by $\mu_{l(s)}$, i.e. by eigenvalues of $H_{||}$ for eigenvectors $|l(s)\rangle$,

$$
\zeta_{+(-)} \;\rightarrow\; \mu_{l(s)} = h_0 + (-)h \equiv m_{l(s)} - \frac{i}{2}\gamma_{l(s)}, \tag{62}
$$

where $m_{l(s)}, \gamma_{l(s)}$ are real, and

$$
\begin{aligned}
h_0 &= \frac{1}{2}(h_{11}+h_{22}), && (63)\\
h &\equiv \sqrt{h_z^2 + h_{12}h_{21}}, && (64)\\
h_z &= \frac{1}{2}(h_{11}-h_{22}). && (65)
\end{aligned}
$$

In the case of neutral kaons these eigenvectors correspond to the long (vector $|l\rangle$) and short (vector $|s\rangle$) living superpositions of K_0 and $\overline{K_0}$.

The following identity taking place for μ_l and μ_s will be needed in next Sections:

$$
\begin{aligned}
\mu_l + \mu_s &= h_{11} + h_{22}, && (66)\\
\mu_s - \mu_l &= 2h \stackrel{\rm def}{\equiv} \Delta\mu, && (67)\\
\mu_l\,\mu_s &= h_{11}h_{22} - h_{12}h_{21} \equiv \det H_{||}, && (68)
\end{aligned}
$$

Using the eigenvectors $|K_{1(2)}\rangle$, (46), of the CP–transformation for the eigenvalues ± 1, vectors $|l\rangle$ and $|s\rangle$ can be expressed as follows [5, 6, 7, 15]

$$
|l(s)\rangle \equiv \frac{1}{\sqrt{1+|\varepsilon_{l(s)}|^2}}\Big(\,|K_{2(1)}\rangle + \varepsilon_{l(s)}|K_{1(2)}\rangle\Big), \tag{69}
$$

where

$$
\begin{aligned}
\varepsilon_l &= \frac{h_{12}-h_{11}+\mu_l}{h_{12}+h_{11}-\mu_l} \equiv -\frac{h_{21}-h_{22}+\mu_l}{h_{21}+h_{22}-\mu_l}, && (70)\\
\varepsilon_s &= \frac{h_{12}+h_{11}-\mu_s}{h_{12}-h_{11}+\mu_l} \equiv -\frac{h_{21}+h_{22}-\mu_s}{h_{21}-h_{22}+\mu_s}, && (71)
\end{aligned}
$$

This form of $|l\rangle$ and $|s\rangle$ is used in many papers when possible departures from CP– or CPT–symmetry in the system considered are discussed. The following parameters are used to describe the scale of CP– and possible CPT – violation effects [5, 6, 7, 15]:

$$\varepsilon \stackrel{\text{def}}{=} \frac{1}{2}(\varepsilon_s + \varepsilon_l), \tag{72}$$

$$\delta \stackrel{\text{def}}{=} \frac{1}{2}(\varepsilon_s - \varepsilon_l). \tag{73}$$

According to the standard interpretation, ε describes violations of CP–symmetry and δ is considered as a CPT–violating parameter [2, 5, 6, 7, 15]. Such an interpretation of these parameters follows from properties of LOY theory of time evolution in the subspace of neutral kaons [1] — [22]. We have

$$\varepsilon = \frac{h_{12} - h_{21}}{D} \tag{74}$$

$$\delta = \frac{h_{11} - h_{22}}{D} \equiv \frac{2h_z}{D}, \tag{75}$$

where

$$D \stackrel{\text{def}}{=} h_{12} + h_{21} + \Delta\mu. \tag{76}$$

Starting from Eqs. (66) — (68) and (70), (71) and using some known identities for μ_l, μ_s one can express matrix elements h_{jk} of $H_{\|}$ in terms of the physical parameters $\varepsilon_{l(s)}$ and $\mu_{l(s)}$:

$$h_{11} = \frac{\mu_s + \mu_l}{2} + \frac{\mu_s - \mu_l}{2}\frac{\varepsilon_s - \varepsilon_l}{1 - \varepsilon_l\varepsilon_s}, \tag{77}$$

$$h_{22} = \frac{\mu_s + \mu_l}{2} - \frac{\mu_s - \mu_l}{2}\frac{\varepsilon_s - \varepsilon_l}{1 - \varepsilon_l\varepsilon_s}, \tag{78}$$

$$h_{12} = \frac{\mu_s - \mu_l}{2}\frac{(1 + \varepsilon_l)(1 + \varepsilon_s)}{1 - \varepsilon_l\varepsilon_s}, \tag{79}$$

$$h_{21} = \frac{\mu_s - \mu_l}{2}\frac{(1 - \varepsilon_l)(1 - \varepsilon_s)}{1 - \varepsilon_l\varepsilon_s}. \tag{80}$$

These relations lead to the following equations

$$h_{11} - h_{22} = \Delta\mu\frac{\varepsilon_s - \varepsilon_l}{1 - \varepsilon_l\varepsilon_s}, \tag{81}$$

$$h_{12} + h_{21} = \Delta\mu\frac{1 + \varepsilon_l\varepsilon_s}{1 - \varepsilon_l\varepsilon_s}, \tag{82}$$

$$h_{12} - h_{21} = \Delta\mu\frac{\varepsilon_s + \varepsilon_l}{1 - \varepsilon_l\varepsilon_s}, \tag{83}$$

Note that relations (77) — (83) are valid for arbitrary values of $\varepsilon_{l(s)}$. From (81) one infers that if $\Delta\mu \neq 0$ then:

$$h_{11} = h_{22} \iff \varepsilon_l = \varepsilon_s. \tag{84}$$

Relation (83) enables us to conclude that parameters ε_l and ε_s need not be small, in order that $\varepsilon = 0$ (74). Indeed, the identity (83) implies that for $\Delta\mu \neq 0$

$$h_{12} = h_{21} \iff \varepsilon_l = -\varepsilon_s, \tag{85}$$

for any values of $|\varepsilon_l|, |\varepsilon_s|$.

It is appropriate to emphasize at this point that all relations (77) — (85) do not depend on a special form of the effective Hamiltonian $H_{\|}$. They are induced by geometric relations between various base vectors in two–dimensional subspace $\mathcal{H}_{\|}$. On the other hand, the interpretation of the above relations depends on the properties of the matrix elements h_{jk} of the effective Hamiltonian $H_{\|}$, i.e., if for example $H_{\|} \neq H_{LOY}$, where H_{LOY} is the LOY effective Hamiltonian, then the interpretation of ε (72) and δ (73) etc., need not be the same for $H_{\|}$ and for H_{LOY}.

Experimentally measured values of parameters $\varepsilon_l, \varepsilon_s$ are very small for neutral kaons. Assuming

$$|\varepsilon_l| \ll 1, \quad |\varepsilon_s| \ll 1, \tag{86}$$

from (81) one finds:

$$h_{11} - h_{22} \simeq (\mu_s - \mu_l)(\varepsilon_s - \varepsilon_l), \tag{87}$$

and (82) implies

$$h_{12} + h_{21} \simeq \mu_s - \mu_l, \tag{88}$$

and (83) gives

$$h_{12} - h_{21} \simeq (\mu_s - \mu_l)(\varepsilon_s + \varepsilon_l). \tag{89}$$

Relation (88) means that in the considered case of small values of parameters $|\varepsilon_l|, |\varepsilon_s|$ (86), the quantity D (76) appearing in the formulae for δ and ε approximately equals

$$D \simeq 2(\mu_s - \mu_l) \equiv 2\Delta\mu. \tag{90}$$

Keeping in mind that $h_{jk} = M_{jk} - \frac{i}{2}\Gamma_{jk}, M_{kj} = M^*_{jk}, \Gamma_{kj} = \Gamma^*_{jk}$ and then starting form Eqs. (87) — (89) and separating real and imaginary parts one can find some useful relations:

$$\begin{aligned}
2\Re\,(M_{12}) &\simeq m_s - m_l, && (91)\\
2\Re\,(\Gamma_{12}) &\simeq \gamma_s - \gamma_l, && (92)\\
2\Im\,(M_{12}) &\simeq -(\gamma_s - \gamma_l)\Big[\,\Im\,(\frac{\varepsilon_s + \varepsilon_l}{2}) + \tan\,\phi_{SW}\,\Re\,(\frac{\varepsilon_s + \varepsilon_l}{2})\Big], && (93)\\
\Im\,(\Gamma_{12}) &\simeq -(\gamma_s - \gamma_l)\Big[\,\tan\,\phi_{SW}\,\Re\,(\frac{\varepsilon_s + \varepsilon_l}{2}) - \mathrm{Im}(\frac{\varepsilon_s + \varepsilon_l}{2})\Big], && (94)
\end{aligned}$$

etc., where $\Re\,(z)$ and $\Im\,(z)$ denote the real and imaginary parts of z respectively, and

$$\tan\,\phi_{SW} \stackrel{\text{def}}{=} \frac{2(m_l - m_s)}{\gamma_s - \gamma_l}. \tag{95}$$

and

$$\begin{aligned}
\Re\,(h_{11}-h_{22}) &\equiv M_{11}-M_{22}=M_1-M_2 \\
&\simeq -(\gamma_s-\gamma_l)\Big[\,\tan\phi_{SW}\,\Re\,(\frac{\varepsilon_s-\varepsilon_l)}{2}) \\
&\quad -\Im\,(\frac{\varepsilon_s-\varepsilon_l)}{2})\Big], \qquad (96) \\
-\Im\,(h_{11}-h_{22}) &\equiv \frac{1}{2}(\Gamma_{11}-\Gamma_{22}) \\
&\simeq (\gamma_s-\gamma_l)\,\Big[\,\Re\,(\frac{\varepsilon_s-\varepsilon_l)}{2}) \\
&\quad +\tan\phi_{SW}\,\Im\,(\frac{\varepsilon_s-\varepsilon_l)}{2})\Big], \qquad (97)
\end{aligned}$$

etc.. One should remember that relations (91) — (94) and (96), (97) are valid only if condition (86) holds. Completing the system of these last six relations one can rewrite Eq. (66) to obtain

$$\begin{aligned}
M_{11}+M_{22} &= m_l+m_s, \qquad (98) \\
\Gamma_{11}+\Gamma_{22} &= \gamma_l+\gamma_s. \qquad (99)
\end{aligned}$$

These last two Equations are exact independently of whether the condition (86) holds or not.

3 Lee, Oehme and Yang model.

3.1 Lee, Oehme and Yang approximation.

The source of The Lee, Oehme and Yang (LOY) approximation for decay of neutral kaons is the well known Weisskopf–Wigner approach to a description of unstable states. Within this approach, the Hamiltonian H for the problem is divided into two parts $H^{(0)}$ and $H^{(1)}$ such that $|K_0\rangle \equiv |\mathbf{1}\rangle$ and $|\overline{K}_0\rangle \equiv |\mathbf{2}\rangle$ are twofold degenerate eigenstates of $H^{(0)}$ to the eigenvalue m_0,

$$H^{(0)}|\mathbf{j}\rangle = m_0|\mathbf{j}\rangle, \;\; j=1,2; \qquad (100)$$

and $H^{(1)} \equiv H - H^{(0)}$ induces transitions from these states to other (unbound) eigenstates $|\varepsilon\rangle$ of $H^{(0)}$ and consequently also between $|K_0\rangle$ and $|\overline{K}_0\rangle$. So, the problem which one usually considers is the time evolution of a state which is prepared initially as a superposition of $|K_0\rangle$ and $|\overline{K}_0\rangle$ states.

In the kaon rest–frame, this time evolution is governed by the Schrödinger equation (2). Solutions, $|\psi;t\rangle$, (3), of this Equation have the form

$$|\psi;t\rangle = a_1(t)|\mathbf{1}\rangle + a_2(t)|\mathbf{2}\rangle + \sum_j F_j(t)|F_j\rangle,$$

where $|F_j\rangle \equiv \sum_\varepsilon \langle\varepsilon|F_j\rangle|\varepsilon\rangle = \sum_\varepsilon f_j(\varepsilon)|\varepsilon\rangle \in \mathcal{H}$ represents the decay products in the channel j; $\langle\varepsilon|\mathbf{k}\rangle = 0$, $k = 1, 2$. It is assumed that $F_j(0) = 0$.

Using the interaction representation and rescaling respectively energy ε: defining $\omega = \varepsilon - m_0$ which means that the zero of energy is taken to be rest energy of K, instead of the Schröinger equation (2) for $|\psi; t\rangle$, Lee, Oehme and Yang obtained the following equations for amplitudes $a_1(t)$, $a_2(t)$ and $F_j(\omega, t)$ replacing $F_j(t)$ [1], [3], (for details see [36]),

$$i\frac{\partial}{\partial t}a_k(t) = \sum_{l=1}^{2} H_{kl}a_l(t) + \sum_{j,\omega} H_{kj}(\omega)F_j(\omega,t)e^{-i\omega t}, \quad (k = 1, 2) \tag{101}$$

$$i\frac{\partial}{\partial t}F_j(\omega,t) = e^{i\omega t}[H_{j1}(\omega)a_1(t) + H_{j2}(\omega)a_2(t)], \tag{102}$$

where $H_{kj}(\omega) = H_{jk}(\omega)^*$, $(k = 1, 2)$, are the matrix elements responsible for the decay and

$$H_{kl} = \langle\mathbf{k}|H|\mathbf{l}\rangle \quad (k, l = 1, 2). \tag{103}$$

Equations (101) are exact, Equation (102) has been obtained by ignoring the series containing matrix elements of type $\langle\varepsilon|H^{(1)}|\varepsilon'\rangle$. (Eq. (102) is exact for the model in which $\langle\varepsilon|H^{(1)}|\varepsilon'\rangle = 0$ — if these matrix elements are very small, then this equation is be treated as a very good approximation for the model considered).

The boundary conditions for Eqs. (101), (102) are the following:

$$a_k(0) \neq 0, \quad (k = 1, 2), \tag{104}$$

and

$$F_j(\omega, 0) = 0. \tag{105}$$

To solve the system of coupled Equations (101), (102), the exponential time dependence for amplitudes $a_k(t)$ has been assumed in [1], i.e., it has been assumed that

$$\frac{a_1(t)}{a_1(0)} = \frac{a_2(t)}{a_2(0)} \equiv e^{-\frac{\Lambda t}{2}}, \quad \Re\,(\Lambda) > 0. \tag{106}$$

This crucial assumption is the essence of the approximation which was made in [1] and determines the properties of the so–called LOY model of neutral kaons decay, i.e., the effective Hamiltonian H_{LOY} governing time evolution in neutral kaons subspace, which is a consequence of Eqs. (101), (102) and of the requirement (106).

Inserting (106) into Eq. (102) and taking into account (105), Eq. (102) can easily be solved to obtain

$$\begin{aligned} iF_j(\omega,t) &= \Big[H_{j1}(\omega)a_1(0) + H_{j2}(\omega)a_2(0)\Big]\frac{e^{i\omega t - \Lambda t/2} - 1}{i\omega - \Lambda/2} \\ &\equiv -i\Big[H_{j1}(\omega)a_1(t) + H_{j2}(\omega)a_2(t)\Big]e^{i\omega t}\,\frac{1 - e^{\Lambda t/2}e^{-i\omega t}}{\omega + i\frac{\Lambda}{2}}. \end{aligned} \tag{107}$$

Next, one can eliminate the $F_j(\omega,t)$ in (101) by substituting (107) back into (101), and then considering Λ as a very small number: $\Lambda \simeq 0$ one finds the following equation for amplitudes $a_k(t)$ (for details, the reader is referred, e.g., to [3, 36])

$$i\frac{\partial}{\partial t}\left(\begin{array}{c} a_1(t) \\ a_2(t) \end{array}\right) = H_{LOY}\left(\begin{array}{c} a_1(t) \\ a_2(t) \end{array}\right), \tag{108}$$

where (see [1] — [22]) $H_{LOY} \equiv M_{LOY} - \frac{i}{2}\Gamma_{LOY}$, and $M_{LOY} = M_{LOY}^{+}$, $\Gamma_{LOY} = \Gamma_{LOY}^{+}$ are (2×2) matrices. Standard formulae for matrix elements $h_{kl}^{LOY} \stackrel{\rm def}{=} \langle \mathbf{k}|H_{LOY}|\mathbf{l}\rangle$ $\equiv M_{kl}^{LOY} - \frac{i}{2}\Gamma_{kl}^{LOY}$ can be found, e.g., in [1] — [22]. One has

$$h_{jk}^{LOY} = H_{jk} - \Sigma_{jk}(m_0) \equiv M_{jk}^{LOY} - \frac{i}{2}\Gamma_{jk}^{LOY}, \;\; (j,k = 1,2), \tag{109}$$

where $\Sigma_{jk}(\varepsilon) = \langle \mathbf{j}|\Sigma(\varepsilon)|\mathbf{k}\rangle$, and

$$\begin{aligned} \Sigma(\varepsilon) &= PHQ\frac{1}{QHQ - \varepsilon - i0}QHP \\ &\stackrel{\rm def}{=} \Sigma^R(\varepsilon) + i\Sigma^I(\varepsilon), \end{aligned} \tag{110}$$

and projectors P, Q are defined formula (19). For ε real one finds $\Sigma^R(\varepsilon) = \Sigma^R(\varepsilon)^+$ and $\Sigma^I(\varepsilon) = \Sigma^I(\varepsilon)^+ \equiv \frac{1}{2}\Gamma(\varepsilon)$. Taking into account (100) one can write that simply

$$\begin{aligned} M_{jk}^{LOY} &= m_0\delta_{jk} + \langle \mathbf{j}|H^{(1)}|\mathbf{k}\rangle - \langle \mathbf{j}|HQ \;\; \text{P.v.}\frac{1}{QHQ - m_0} \;\; HQ|\mathbf{k}\rangle, \\ \Gamma_{jk}^{LOY} &= 2\pi\langle \mathbf{j}|HQ\;\delta(QHQ - m_0)\;QH|\mathbf{k}\rangle, \end{aligned}$$

(here P.v. denotes a principal value). These formulae are the frame for almost all calculations of parameters characterizing CP–violation effects in neutral kaons decays, and for searching for possible violations of CPT–symmetry, and for designing CPT–violation tests in such a system [2] — [22]. The compact, operator form of H_{LOY} is

$$H_{LOY} \equiv PHP - \Sigma(m_0). \tag{111}$$

H_{LOY} acts in the subspace $\mathcal{H}_{\|} \stackrel{\rm def}{=} P\mathcal{H}$ of $\mathcal{H}$ — in the subspace of unstable states $|\mathbf{1}\rangle, |\mathbf{2}\rangle \in \mathcal{H}_{\|}$. The subspace of decay products $\mathcal{H}_{\perp}$ is defined by the projector Q: $\mathcal{H}_{\perp} \stackrel{\rm def}{=} Q\mathcal{H} \ni |F_j\rangle$.

Using H_{LOY} solutions of the evolution equation (108) can be written by means of an evolution operator $U_{\|}^{LOY}(t)$ for subspace $\mathcal{H}_{\|}$ as follows

$$\left(\begin{array}{c} a_1(t) \\ a_2(t) \end{array}\right) = U_{\|}^{LOY}(t)\left(\begin{array}{c} a_1(0) \\ a_2(0) \end{array}\right), \tag{112}$$

or

$$|\psi; t\rangle_{\|} = U_{\|}^{LOY}(t)|\psi\rangle_{\|}, \tag{113}$$

where

$$
\begin{aligned}
|\psi;t\rangle_{\|} &\stackrel{\text{def}}{=} P|\psi;t\rangle \equiv a_1(t)|\mathbf{1}\rangle + a_2(t)|\mathbf{2}\rangle, \\
|\psi\rangle_{\|} &= |\psi; t=0\rangle_{\|},
\end{aligned} \tag{114}
$$

and

$$
\begin{aligned}
U_{\|}^{LOY} &= \exp(-itH_{LOY}) \\
&\equiv e^{-it\,h_0^{LOY}}\Big[I_{\|}\cos(t\,h^{LOY}) - i\frac{\vec{h}^{\,LOY} \bullet \vec{\sigma}}{h^{LOY}}\sin(t\,h^{LOY})\Big].
\end{aligned} \tag{115}
$$

Here the Pauli matrices representation is used (see (24) — (28)):

$$
H_{LOY} \equiv h_0^{LOY} I_{\|} + \vec{h}^{\,LOY} \bullet \vec{\sigma}, \tag{116}
$$

$$
h_0^{LOY} = \frac{1}{2}[h_{11}^{LOY} + h_{22}^{LOY}],
$$

$$
h_z^{LOY} = \frac{1}{2}[h_{11}^{LOY} - h_{22}^{LOY}],
$$

$$
\begin{aligned}
(h^{LOY})^2 \equiv \vec{h}^{\,LOY} \bullet \vec{h}^{\,LOY} &= (h_x^{LOY})^2 + (h_y^{LOY})^2 + (h_z^{LOY})^2 \\
&\equiv h_{12}^{LOY} h_{21}^{LOY} + (h_z^{LOY})^2.
\end{aligned}
$$

3.2 Properties of neutral kaons in the case of conserved CP–symmetry.

Assuming that CP–symmetry is conserved in the system considered, i.e., that relation (36) is valid, one finds that

$$
[\mathcal{CP}, PHP] = [\mathcal{CP}, \Sigma(m_0)] = 0, \tag{117}
$$

which means that

$$
[\mathcal{CP}, H_{LOY}] = 0. \tag{118}
$$

Using this property and (44), the matrix elements M_{jk}^{LOY} and Γ_{jk}^{LOY} can be found as

$$
M_{11}^{LOY} = M_{22}^{LOY}, \;\; \Gamma_{11}^{LOY} = \Gamma_{22}^{LOY}, \tag{119}
$$

$$
M_{12}^{LOY} = M_{21}^{LOY}, \;\; \Gamma_{12}^{LOY} = \Gamma_{21}^{LOY}, \tag{120}
$$

which give

$$
h_{11}^{LOY} = h_{22}^{LOY} \equiv h_0^{LOY}, \;\; h_{12}^{LOY} = h_{21}^{LOY}. \tag{121}
$$

These relations have the following consequences for eigenvectors and eigenvalues of H_{LOY} in the case of conserved CP–symmetry:

$$
\alpha_{-(+)} \equiv \alpha_{2(1)} = -(+)1, \tag{122}
$$

$$|e_{\pm}\rangle \longrightarrow |K_{1(2)}\rangle \rho_{1(2)} \Big(|\mathbf{1}\rangle - \alpha_{1(2)} |\mathbf{2}\rangle \Big) \equiv 2^{-1/2} \Big(|\mathbf{1}\rangle - (+) |\mathbf{2}\rangle \Big), \tag{123}$$

$$\zeta_{-(+)} \longrightarrow m_{1(2)}^{LOY} = h_0^{LOY} - (+)(h_{12}^{LOY} h_{21}^{LOY})^{1/2}, \tag{124}$$

and this is the picture which one observes within the LOY theory in the case of conserved CP symmetry.

3.3 The case of nonconserved CP and conserved CPT.

It is known that in 1964 it was announced that long living K_2 states exhibited a decay into two pions, forbidden in the case of conserved CP–symmetry [32]. These CP–violating decays for the $K_0, \overline{K}_0$ complex are only of order 0.1 % of the CP–conserving decays but nevertheless this is the proof that CP–symmetry is not conserved in neutral kaons complex. So, the only possibility is to assume that CP–symmetry is violated but CPT–symmetry is conserved. This is because within the context of local quantum field theory, CPT conservation is a theorem.

Assuming that CPT is the symmetry for the system under investigation and that subspaces of neutral kaons $\mathcal{H}_{\|}$ and their decay products $Q\mathcal{H} \equiv \mathcal{H}_{\perp}$ are invariant under CPT–transformation, i.e., assuming (37), (38), one easily finds that $H_{11} \equiv H_{22}$, and

$$\Theta \Sigma(m_0) \Theta^{-1} = \Sigma^{+}(m_0), \tag{125}$$

i.e.,

$$\Theta H_{LOY} \Theta^{-1} = H_{LOY}^{+}, \tag{126}$$

which implies

$$\Sigma_{11}^{R}(m_0) \equiv \Sigma_{22}^{R}(m_0), \quad \Sigma_{11}^{I}(m_0) \equiv \Sigma_{22}^{I}(m_0), \tag{127}$$

and

$$\Sigma_{21}^{R}(m_0) \equiv [\Sigma_{12}^{R}(m_0)]^{*}, \quad \Sigma_{21}^{I}(m_0) \equiv [\Sigma_{12}^{I}(m_0)]^{*}, \tag{128}$$

Therefore the obvious conclusion, exploited widely in the literature, is that

$$M_{11}^{LOY} = M_{22}^{LOY} \stackrel{\text{def}}{=} M_0^{LOY}, \;\; \Gamma_{11}^{LOY} = \Gamma_{22}^{LOY} \stackrel{\text{def}}{=} \Gamma_0^{LOY}, \tag{129}$$

or,

$$\Sigma_{11}(m_0) \equiv \Sigma_{22}(m_0), \tag{130}$$

which means that

$$h_{11}^{LOY} \equiv h_{22}^{LOY} \stackrel{\text{def}}{=} h_0^{LOY}, \tag{131}$$

and

$$M_{21}^{LOY} = (M_{12}^{LOY})^{*}, \;\; \Gamma_{21}^{LOY} = (\Gamma_{12}^{LOY})^{*}, \tag{132}$$

in CPT–invariant system (30), (38). Relations (129) and (131) are the standard result of the LOY approach and this is the picture which one meets in the literature [2] — [22] and which one obtains searching only for properties of matrix elements of above obtained H_{LOY}.

These properties mean that the effective Hamiltonian $H_{LOY} \equiv H_{LOY}^{\Theta}$ (H_{LOY}^{Θ} denotes the operator H_{LOY} when the property (37) occurs) is represented by the following (2×2) matrix

$$H_{LOY}^{\Theta} \equiv \begin{pmatrix} M_0 - \frac{i}{2}\Gamma_0, & M_{12}^{LOY} - \frac{i}{2}\Gamma_{12}^{LOY} \\ M_{12}^{LOY\,*} - \frac{i}{2}\Gamma_{12}^{LOY\,*} & M_0 - \frac{i}{2}\Gamma_0 \end{pmatrix}, \tag{133}$$

and lead to the following form of eigenvectors and eigenvalues of H_{LOY}^{Θ}. From (35) and (131) it follows that

$$|e_{+(-)}\rangle \longrightarrow |K_{l(s)}\rangle = \rho_{l(s)}^{LOY} \Big(|\mathbf{1}\rangle - \alpha_{l(s)}^{LOY} |\mathbf{2}\rangle \Big) \equiv \rho_{l(s)}^{LOY} \Big(|\mathbf{1}\rangle + (-)a|\mathbf{2}\rangle \Big), \tag{134}$$

where

$$\alpha_{+(-)} \equiv \alpha_{l(s)}^{LOY} \stackrel{\text{def}}{=} -(+)a \equiv -(+)\Big(\frac{h_{21}^{LOY}}{h_{12}^{LOY}}\Big)^{1/2}, \tag{135}$$

$$|a| \neq 1, \tag{136}$$

and

$$|\rho_{l(s)}^{LOY}|^2 = (1 + |a|^2\,)^{-1}. \tag{137}$$

We have also that

$$\zeta_{+(-)} \longrightarrow \mu_{l(s)}^{LOY} = h_0^{LOY} + (-)h^{LOY}, \tag{138}$$

$$h^{LOY} = (h_{12}^{LOY} h_{21}^{LOY})^{1/2}. \tag{139}$$

Let us notice that in this case

$$\langle K_s|K_l\rangle \neq 0, \tag{140}$$

in contradistinction to the case of conserved CP–symmetry, where $\langle K_1|K_2\rangle = 0$ (47). States $|K_l\rangle$, $|K_s\rangle$ are long and short living superpositions of K_0 and $\overline{K}_0$ respectively. Experimentally determined life-times are $\tau_l \simeq 5.17 \times 10^{-8}$ s, $\tau_s \simeq 0.89 \times 10^{-10}$ s, which mean that $c\tau_l \simeq 15.49$ m and $c\tau_s \simeq 2.68$ cm. There is $A(K \rightarrow 2\pi) \neq -A(\overline{K} \rightarrow 2\pi)$ in this case and therefore two– and three–pion decays are allowed for K_l and K_s both.

3.4 The case of nonconserved CPT.

If to assume that CPT–symmetry is not conserved in the physical system under consideration, i.e., that

$$[\Theta, H] \neq 0, \tag{141}$$

then $h_{11}^{LOY} \neq h_{22}^{LOY}$, which imply that $|\alpha_l| \neq |\alpha_s|$ in expansions (34), (59). It is convenient to express difference between H_{LOY}^{Θ} and the effective Hamiltonian H_{LOY}

appearing within the LOY approach in the case of nonconserved CPT–symmetry as follows

$$
\begin{aligned}
H_{LOY} &\equiv H_{LOY}^{\Theta} + \delta H_{LOY} \\
&= \begin{pmatrix} (M_0 + \frac{1}{2}\delta M) - \frac{i}{2}(\Gamma_0 + \frac{1}{2}\delta\Gamma), & M_{12}^{LOY} - \frac{i}{2}\Gamma_{12}^{LOY} \\ M_{12}^{LOY\,*} - \frac{i}{2}\Gamma_{12}^{LOY\,*} & (M_0 - \frac{1}{2}\delta M) - \frac{i}{2}(\Gamma_0 - \frac{1}{2}\delta\Gamma) \end{pmatrix}.
\end{aligned} \tag{142}
$$

Within the LOY model the δM and $\delta\Gamma$ terms violate CPT–symmetry.

Generally, when $[\mathcal{CP}, H] \neq 0$, $[\Theta, H] \neq 0$, the eigenvectors $|l\rangle, |s\rangle$ of $H_{\parallel}$ for the eigenvalues $\mu_{l(s)}$ differ from $|K_{1(2)}\rangle$ (123) and $|K_{l(s)}\rangle$ (134), and are not orthogonal [5] — [22], [32] — [35].

It is convenient to express the CP– and CPT–violation parameters in relation to the orthogonal eigenvectors $|K_1\rangle$ and $|K_2\rangle$ of the $\mathcal{CP}$–transformation for the eigenvalues ∓ 1 (123). Vectors $|l\rangle$, $|s\rangle$ written down in the $|K_1\rangle$, $|K_2\rangle$ basis have the form (69) [7, 6, 15]. Usually, instead of the parameters ε_l, ε_s, the parameters, ε, (72) and δ, (73), are used [5, 7, 15, 16].

The interpretation of δ as the CPT–violating parameter follows directly from a formula (75) for this parameter, when one inserts into this formula matrix elements h_{jk}^{LOY} of the effective Hamiltonian $H_{\parallel} \equiv H_{LOY}$: see (109), (29). The relation (131) leads to the conclusion

$$
\delta \cong \delta^{LOY} = 0, \tag{143}
$$

where $\delta^{LOY} \equiv \delta(h_{jk} \equiv h_{jk}^{LOY})$.

The parameter δ (or equivalent parameters) is usually measured in experimental tests of CPT invariance in the neutral kaon system [3] — [5], [7, 15, 18]. The standard interpretation of the result of this experiment (which is a straightforward consequence of (143)) is that the result the result $\delta \cong 0$ means that the property $[\Theta, H] = 0$ holds in the investigated system, while the opposite result, $\delta \neq 0$, means that the CPT—symmetry is not conserved in this system – this again follows from the traditional interpretation which may be based on (37), (143). Indeed, relations (143) and (131) mean that (see (73))

$$
\varepsilon_l - \varepsilon_s = 0. \tag{144}
$$

Therefore the tests based on the relation (96) are considered as the test of CPT–invariance and the results of such tests are interpreted that the masses of the particle "1" (the K_0 meson) and its antiparticle "2" (the $\overline{K}_0$ meson) must be equal if CPT–symmetry holds. Parameters $\varepsilon_l, \varepsilon_s, \gamma_l, \gamma_s$, etc., appearing in the right side of the relation (96) can be extracted from experiments in such tests and then these parameters can be used to estimate the left side of this relation. The estimation for the mass difference obtained in this way with the use of the recent data [18] reads

$$
\frac{|M_1 - M_2|}{m_{K_0}} = \frac{|m_{K_0} - m_{\overline{K}_0}|}{m_{K_0}} \leq 10^{-18}, \tag{145}
$$

and this estimation is considered as indicating no CPT–violation effect. This interpretation follows from the properties of the H_{LOY}.

Summing up, according to the above, physicists believe that

$$\delta = 0 \Leftrightarrow |M_1 - M_2| = 0 \quad \Rightarrow \quad [\Theta, H] = 0\,(?), \tag{146}$$
$$\delta \neq 0 \Leftrightarrow |M_1 - M_2| \neq 0 \quad \Rightarrow \quad [\Theta, H] \neq 0, (?). \tag{147}$$

This is the standard result of the LOY approach and this is the picture which one meets in the literature [1] — [22].

4 Real properties of time evolution in subspace of neutral kaons.

The aim of this Section is to show that the diagonal matrix elements of the exact effective Hamiltonian $H_{||}$ can not be equal when the total system under consideration is CPT invariant but CP noninvariant.

Universal properties of the (unstable) particle–antiparticle subsystem of the system described by the Hamiltonian H, for which the relation (37) holds, can be extracted from the matrix elements of the exact $U_{||}(t)$ appearing in (22). Such $U_{||}(t)$ has the following form

$$U_{||}(t) = PU(t)P, \tag{148}$$

where P is given by the formula (19) and $U(t)$ is the total unitary evolution operator, which solves the Schrödinger equation (2). Operator $U_{||}(t)$ acts in the subspace of unstable states $\mathcal{H}_{||} \equiv P\mathcal{H}$. Of course, $U_{||}(t)$ has nontrivial form only if

$$[P, H] \neq 0, \tag{149}$$

and only then transitions of states from $\mathcal{H}_{||}$ into $\mathcal{H}_{\perp}$ and vice versa, i.e., decay and regeneration processes, are allowed.

Using the matrix representation one finds

$$U_{||}(t) \equiv \left(\begin{array}{cc} \mathbf{A}(t) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array} \right) \tag{150}$$

where $\mathbf{0}$ denotes the suitable zero submatrices and a submatrix $\mathbf{A}(t)$ is the 2×2 matrix acting in $\mathcal{H}_{||}$

$$\mathbf{A}(t) = \left(\begin{array}{cc} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{array} \right) \tag{151}$$

and $A_{jk}(t)$ is given by (6) for $|\psi\rangle = |\mathbf{k}\rangle$, $(j, k = 1, 2)$.

Now assuming (37) and using, e.g., the phase convention defined by the formula (45) one easily finds that [11] — [14], [37]

$$A_{11}(t) = A_{22}(t). \tag{152}$$

Note that assumptions (37) and (45) give no relations between $A_{12}(t)$ and $A_{21}(t)$.

The important relation between amplitudes $A_{12}(t)$ and $A_{21}(t)$ follows from the famous Khalfin's Theorem [11] — [14], [38]. This Theorem states that in the case of unstable states, if amplitudes $A_{12}(t)$ and $A_{21}(t)$ have the same time dependence

$$r(t) \stackrel{\text{def}}{=} \frac{A_{12}(t)}{A_{21}(t)} = \text{const} \equiv r, , \tag{153}$$

then it must be $|r| = 1$.

For unstable particles the relation (152) means that decay laws

$$p_j(t) \stackrel{\text{def}}{=} |A_{jj}(t)|^2, \tag{154}$$

(where $j = 1, 2$), of the particle $|\mathbf{1}\rangle$ and its antiparticle $|\mathbf{2}\rangle$ are equal,

$$p_1(t) \equiv p_2(t). \tag{155}$$

The consequence of this last property is that the decay rates of the particle $|\mathbf{1}\rangle$ and its antiparticle $|\mathbf{2}\rangle$

$$\gamma_j(t) \stackrel{\text{def}}{=} -\frac{1}{p_j(t)} \frac{\partial p_j(t)}{\partial t},$$

must be equal too,

$$\gamma_1(t) = \gamma_2(t), \tag{156}$$

On the other hand from (152) it does not follow that the masses of the particle "1" and the antiparticle "2" should be equal.

More conclusions about the properties of the matrix elements of $H_{||}$, that is in particular about M_{jj}, one can infer analyzing the following identity [23], [39] — [41]

$$H_{||} \equiv H_{||}(t) = i \frac{\partial U_{||}(t)}{\partial t} [U_{||}(t)]^{-1}, \tag{157}$$

where $[U_{||}(t)]^{-1}$ is defined as follows

$$U_{||}(t) \, [U_{||}(t)]^{-1} = [U_{||}(t)]^{-1} \, U_{||}(t) \; = \; P. \tag{158}$$

(Note that the identity (157) holds, independent of whether $[P, H] \neq 0$ or $[P, H] = 0$). The expression (157) can be rewritten using the matrix $\mathbf{A}(t)$

$$H_{||}(t) \equiv i \frac{\partial \mathbf{A}(t)}{\partial t} [\mathbf{A}(t)]^{-1}. \tag{159}$$

Relations (157), (159) must be fulfilled by the exact as well as by every approximate effective Hamiltonian governing the time evolution in every two dimensional subspace $\mathcal{H}_{||}$ of states $\mathcal{H}$ [23, 39] — [41].

It is easy to find from (157) the general formulae for the diagonal matrix elements, h_{jj}, of $H_{||}(t)$, in which we are interested. We have [42]

$$h_{11}(t) = \frac{i}{\det \mathbf{A}(t)} \Big(\frac{\partial A_{11}(t)}{\partial t} A_{22}(t) - \frac{\partial A_{12}(t)}{\partial t} A_{21}(t) \Big), \tag{160}$$

$$h_{22}(t) = \frac{i}{\det \mathbf{A}(t)} \Big(- \frac{\partial A_{21}(t)}{\partial t} A_{12}(t) + \frac{\partial A_{22}(t)}{\partial t} A_{11}(t) \Big). \tag{161}$$

Now, assuming (37) and using the consequence (152) of this assumption, one finds

$$h_{11}(t) - h_{22}(t) = \frac{i}{\det \mathbf{A}(t)} \Big(\frac{\partial A_{21}(t)}{\partial t} A_{12}(t) - \frac{\partial A_{12}(t)}{\partial t} A_{21}(t) \Big). \tag{162}$$

Next, after some algebra one obtains

$$h_{11}(t) - h_{22}(t) = -i \, \frac{A_{12}(t)\, A_{21}(t)}{\det \mathbf{A}(t)} \, \frac{\partial}{\partial t} \ln \Big(\frac{A_{12}(t)}{A_{21}(t)} \Big). \tag{163}$$

This result means that in the considered case for $t > 0$ the following Theorem holds [42]:

Theorem 1.

$$h_{11}(t) - h_{22}(t) = 0 \;\; \Leftrightarrow \;\; \frac{A_{12}(t)}{A_{21}(t)} \; = \; \text{const.}, \;\; (t > 0). \tag{164}$$

Thus for $t > 0$ the problem under studies is reduced to the Khalfin's Theorem (see the relation (153)).

From (160) and (161) it is easy to see that at $t = 0$

$$h_{jj}(0) = \langle \mathbf{j} | H | \mathbf{j} \rangle, \;\; (j = 1, 2), \tag{165}$$

which means that in a CPT invariant system (37) in the case of pairs of unstable particles, for which transformations of type (45) hold

$$M_{11}(0) = M_{22}(0) \equiv \langle \mathbf{1} | H | \mathbf{1} \rangle, \tag{166}$$

the unstable particles "1" and "2" are created at $t = t_0 \equiv 0$ as particles with equal masses. The same result can be obtained from the formula (163) by taking $t \to 0$.

In the general case

$$h_{jk}(0) = H_{jk}, \quad (j, k = 1, 2). \tag{167}$$

Now let us go on to analyze the conclusions following from the Khalfin's Theorem. CP noninvariance requires that $|r| \neq 1$ [11, 13, 14, 37] (see also [1] — [8], [11]

— [22]). This means that in such a case it must be $r = r(t) \neq$ const.. So, if in the system considered the property (37) holds but

$$[\mathcal{CP}, H] \neq 0, \tag{168}$$

and the unstable states "1" and "2" are connected by a relation of type (45), then at $t > 0$ it must be $(h_{11}(t) - h_{22}(t)) \neq 0$ in this system.

On the other hand to complete the discussion of the problem one can examine consequences of the assumptions that $(h_{11}(t) - h_{22}(t)) = 0$ is admissible for $t > 0$ and that the system under considerations need not be CP– or CPT–invariant. In such a case an analysis of the considerations leading to the Theorem 1 allows one to conclude that

Conclusion 1.
If $(h_{11}(t) - h_{22}(t)) = 0$ for $t > 0$ then it must be
a)

$$\frac{A_{11}(t)}{A_{22}(t)} = \text{const.}, \quad \text{and} \quad \frac{A_{12}(t)}{A_{21}(t)} = \text{const.}, \quad \text{for} \quad (t > 0),$$

or,
b)

$$\frac{A_{11}(t)}{A_{22}(t)} \neq \text{const.}, \quad \text{and} \quad \frac{A_{12}(t)}{A_{21}(t)} \neq \text{const.}, \quad \text{for} \quad (t > 0).$$

The case a) means that CP–symmetry is conserved and there is no any information about CPT invariance. The case b) denotes that system under considerations is neither CP–invariant nor CPT–invariant.

Now let us examine properties of $\Re\,(h_{11}(t) - h_{22}(t))$ for $t > t = t_0 \equiv 0$. It can be done, e.g., using the methods exploited in [43].

In the nontrivial case (149) from (157), using (2), (5) and (148) we find

$$\begin{aligned} H_{\parallel}(t) &\equiv PHU(t)P[U_{\parallel}(t)]^{-1}P && (169)\\ &\equiv PHP + PHQU(t)[U_{\parallel}(t)]^{-1}P && (170)\\ &\stackrel{\text{def}}{\equiv} PHP + V_{\parallel}(t). && (171) \end{aligned}$$

Thus [44, 45]

$$H_{\parallel}(0) \equiv PHP, \;\; V_{\parallel}(0) = 0, \;\; V_{\parallel}(t \to 0) \simeq -itPHQHP, \tag{172}$$

so, in general $H_{\parallel}(0) \neq H_{\parallel}(t \gg t_0 = 0)$ and $V_{\parallel}(t \neq 0) \neq V_{\parallel}^{+}(t \neq 0)$, $H_{\parallel}(t \neq 0) \neq H_{\parallel}^{+}(t \neq 0)$.

Eigenvectors of time–dependent $H_{\|}(t)$ can depend on time t: From (34) we obtain

$$|e_{+}\rangle \longrightarrow |l^{t}\rangle, \qquad |e_{-}\rangle \longrightarrow |s^{t}\rangle, \tag{173}$$

and, in general, they are not orthogonal. In long time region ($t \to \infty$) vectors $|l(s)^{t\to\infty}\rangle$ correspond to the states $|K_{l(s)}\rangle$ (134) obtained within the LOY approach. In short time region, from (172) we have (167) and

$$h_{jk}(t \to 0) = H_{jk} - it\langle \mathbf{j}|HQH|\mathbf{k}\rangle, \tag{174}$$

This implies that in the case considered

$$\alpha_{l(s)}(t \to 0) = -(+)\Big(\frac{H_{21}}{H_{12}}\Big)^{1/2}, \tag{175}$$

if

$$H_{12} \equiv \langle \mathbf{1}|H|\mathbf{2}\rangle = H_{21}^{*} \neq 0. \tag{176}$$

So, in such a case

$$|\rho_{l(s)}(0)|^{2} = \frac{1}{2}. \tag{177}$$

Therefore the eigenvectors for $H_{\|}(t)$ take the following form in the early time period:

$$|l(s)^{t\to 0}\rangle = \rho_{l(s)}(0)\Big[|\mathbf{1}\rangle + (-)\Big(\frac{H_{21}}{H_{12}}\Big)^{1/2}|\mathbf{2}\rangle\Big]. \tag{178}$$

They are orthogonal quite independently of whether CP– or CPT–symmetries are conserved or not.

Beyond the short time region, for $t > 0$, the vectors $|l(s)^{t}\rangle$ are orthogonal only in the case of conserved CP–symmetry. Indeed, if property (36) holds one has

$$[\mathcal{CP}, U_{\|}(t)] = 0, \tag{179}$$

and thus, by the identity (169),

$$[\mathcal{CP}, H_{\|}(t)] = 0, \tag{180}$$

which and definition (44) lead to the following relations

$$h_{11}(t) = h_{22}(t), \qquad h_{12}(t) = h_{21}(t), \tag{181}$$

valid for every t in the considered case of conserved CP. This means that eigenvectors for $H_{\|}(t)$ (157) must be equal $|K_{1}\rangle$ and $|K_{2}\rangle$ (123), i.e., effective Hamiltonians $H_{\|}(t)$ and H_{LOY} lead to the same solutions of the eigenvalue problem in the case of conserved CP–symmetry.

Now let us pass on to considerations of CPT–transformation properties of $H_{\|}$. There is only one assumption for the operator Θ describing CPT–transformation in

$\mathcal{H}$: we require the assumption (38) for Θ to be fulfilled, and, in contradistinction to (37), there is not any assumptions for $[\Theta, H]$. Using this assumption and the identity (169), after some algebra, one finds [46]

$$[\Theta, H_{\|}(t)] = \mathcal{A}(t) + \mathcal{B}(t), \tag{182}$$

where:

$$\begin{aligned} \mathcal{A}(t) &= P[\Theta, H]U(t)P(U_{\|}(t))^{-1}P, && (183)\\ \mathcal{B}(t) &= \Big\{PHQ - PHU(t)P(U_{\|}(t))^{-1}P\Big\}[\Theta, U(t)]P(U_{\|}(t))^{-1}P \\ &\equiv P\Big\{H \; - \; H_{\|}(t)P\Big\}[\Theta, U(t)]P(U_{\|}(t))^{-1}P. && (184)\end{aligned}$$

We observe that $\mathcal{A}(0) \equiv P[\Theta, H]P$ and $\mathcal{B}(0) \equiv 0$. From definitions and general properties of operators $\mathcal{C}$,$\mathcal{P}$ and $\mathcal{T}$ [8], [47] — [50] it is known that $\mathcal{T}U(t{\neq}0) = U_T^{+}(t{\neq}0)\mathcal{T} \neq U(t{\neq}0)\mathcal{T}$ (Wigner's definition for $\mathcal{T}$ is used) [49], and thereby $\Theta U(t \neq 0) = U_{CPT}^{+}(t \neq 0)\Theta$ [47, 48, 49], i.e. $[\Theta, U(t \neq 0)] \neq 0$. So, the component $\mathcal{B}(t)$ in (33) is nonzero for $t \neq 0$ and it is obvious that there is a chance for Θ–operator to commute with the effective Hamiltonian $H_{\|}(t \neq 0)$ only if $[\Theta, H] \neq 0$. On the other hand, the property $[\Theta, H] \neq 0$ does not imply that $[\Theta, H_{\|}(0)] = 0$ or $[\Theta, H_{\|}(0)] \neq 0$. These two possibilities are admissible, but if $[\Theta, H] = 0$ then there is only one possibility: $[\Theta, H_{\|}(0)] = 0$ [43].

From (182) we find

$$\Theta H_{\|}(t)\Theta^{-1} - H_{\|}(t) \equiv (\mathcal{A}(t) + \mathcal{B}(t))\Theta^{-1}. \tag{185}$$

These relations and (169 lead to the conclusion that generally $\Theta H_{\|}(t{\neq}0)\Theta^{-1} \neq H_{\|}^{+}(t{\neq}0)$, and $\Theta H_{\|}(t \neq 0)\Theta^{-1} \neq H_{\|}(t{\neq}0)$.

Now, keeping in mind that $|\mathbf{2}\rangle \equiv |\overline{K}_0\rangle$ is the antiparticle for $|\mathbf{1}\rangle \equiv |K_0\rangle$ and that, by definition, the Θ–operator transforms $|\mathbf{1}\rangle$ in $|\mathbf{2}\rangle$ (39), (the phase convention (45) is assumed) and taking into account another properties of Θ, we obtain from (185)

$$h_{11}(t)^{*} - h_{22}(t) = -\langle \mathbf{2}|(\mathcal{A}(t) + \mathcal{B}(t))|\mathbf{1}\rangle. \tag{186}$$

Adding together expression (186) and its complex conjugate one yields

$$\Re \; (h_{11}(t) - h_{22}(t)) = -\Re \; \langle \mathbf{2}|(\mathcal{A}(t) + \mathcal{B}(t))|\mathbf{1}\rangle. \tag{187}$$

Now, let us assume for a moment that the property (37) occurs, i.e., that $[\Theta, H] = 0$. Then $\mathcal{A}(t) \equiv 0$ and thus $[\Theta, H_{\|}(0)] = 0$, which is in agreement with an earlier, similar result [43]. In this case we have $\Theta U(t) = U^{+}(t)\Theta$, which gives $\Theta U_{\|}(t) = U_{\|}^{+}(t)\Theta$, $\Theta U_{\|}^{-1}(t) = (U_{\|}^{+}(t))^{-1}\Theta$, and

$$[\Theta, U(t)] = -2i(\Im \; U(t))\Theta. \tag{188}$$

This relation leads to the following result in the case of the conserved $\mathcal{CPT}$-symmetry under consideration

$$\mathcal{B}(t) = -2iP\Big\{H \ - \ H_{\|}(t)\ P\Big\}(\Im\ U(t))P(U_{\|}^{+}(t))^{-1}\Theta. \tag{189}$$

From (189) we obtain

$$\langle \mathbf{2}|\mathcal{B}(t)|\mathbf{1}\rangle \equiv 2i\langle \mathbf{2}|(H \ - \ H_{\|}(t)\ P)(\Im\ U(t))P(U_{\|}^{+}(t))^{-1}|\mathbf{2}\rangle. \tag{190}$$

This expression allows us to conclude that $\langle \mathbf{2}|\mathcal{B}(0)|\mathbf{1}\rangle = 0$ and $\Re\,\langle \mathbf{2}|\mathcal{B}(t{\neq}0)|\mathbf{1}\rangle \neq 0$, $\Im\,\langle \mathbf{2}|\mathcal{B}(t{\neq}0)|\mathbf{1}\rangle \neq 0$ if condition (37) holds. This means that in this case it must be $\Re\,(h_{11}(t) - h_{22}(t)) \neq 0$ for $t \neq 0$.

So, there is no possibility for δ to take the value zero for $t > 0$ in the case of conserved CPT–symmetry in the system considered: it must be $\delta \neq 0$ in such a case.

The only possibility for δ to equal zero is if the nonzero contribution of $\mathcal{B}(t \neq 0)$ into $(h_{11}(t) - h_{22}(t))$ is compensated by a nonzero contribution of $\mathcal{A}(t)$. It can be observed that $\Re\,(\langle \mathbf{2}|\mathcal{B}(t{\neq}0)|\mathbf{1}\rangle) \neq 0$ irrespective of whether Θ commutes with H or not, but $\mathcal{A}(t) \neq 0$ only appears if $[\Theta, H] \neq 0$. So, definition (75) of the parameter δ, properties (182), (187) and consequences of (190) lead to the following conclusions for $\delta \equiv \delta(h_{jk}(t \gg t_0 \equiv 0))$:

Conclusion 2.:
a) If $\delta = 0$ then it follows that $[\Theta, H] \neq 0$,
b) If $[\Theta, H] = 0$ then it follows that $\delta \neq 0$.
c) If $\delta \neq 0$ then the cases $[\Theta, H] \neq 0$ or $[\Theta, H] = 0$ are both possible.

The same conclusions are valid also when one uses a density matrix approach for a description of time evolution in K_0, $\overline{K}_0$ complex (see [51]). All results and conclusions of this Section are, in fact, model independent, i.e., they do not depend on the given Hamiltonian H of the system considered. This Section describes general properties of two-state complex (subsystem) evolving in time in two–dimensional subspace $\mathcal{H}_{\|}$ of the total state space of the system $\mathcal{H}$ and interacting with the rest of the system considered.

Assuming the LOY interpretation of $\Re\,(h_{jj}(t))$, $(j = 1, 2)$, one can conclude from the Khalfin's Theorem and from the property (164) that if $A_{12}(t), A_{21}(t) \neq 0$ for $t > 0$ and if the total system considered is CPT–invariant, but CP–noninvariant, then $M_{11}(t) \neq M_{22}(t)$ for $t > 0$, that is, that contrary to the case of stable particles (the bound states), the masses of the simultaneously created unstable particle "1" and its antiparticle "2", which are connected by the relation (45), need not be equal for $t > t_0 = 0$. Of course, such a conclusion contradicts the standard LOY result (129), (131). However, one should remember that the LOY description of neutral K mesons and similar complexes is only an approximate one, and that the

LOY approximation is not perfect. On the other hand the relation (164) and the Khalfin's Theorem follow from the basic principles of the quantum theory and are rigorous. Consequently, their implications should also be considered rigorous.

One should remember that all the above conclusions (as well as the conclusions following from theories based on the effective Hamiltonian obtained within LOY approach) are valid if the experimenter is able to prepare the tested system such that its initial state fulfills condition (4), (17) (or (104) and (105)).

5 CPT theorem and exponential decay.

The aim of this Section is to investigate the consequences of the main assumption of the LOY theory, (106), i. e., the assumption that the decay law of neutral kaons has an exponential form. More precisely, the question if the model assuming an exponential decay law is able to describe correctly CPT symmetry properties of the real system will be discussed.

Assumptions of the CPT Theorem are usually formulated in terms of Wightman functions $W^{(n)}$ [31], [26] — [30]. These will be considered briefly below (details can be found, e.g. in [26] — [31]). If we take, for example, a set of neutral scalar fields $\{\Phi_{\Xi}(x)\} \ni \phi_{\rm A}(x)$, $\rm A \in \Xi$, $x \equiv (ct, \vec{r}) \in M_4$ (M_4 is the Minkowski space-time, so $x^2 \equiv c^2t^2 - \vec{r}^2$), — for the sake of simplicity only such fields will be considered here — Wightman functions are vacuum expectation values for products of n field functions

$$W^{(n)}_{\rm ABC\ldots}(x_1, x_2, \ldots, x_n) \equiv \langle\Omega|\phi_{\rm A}(x_1)\phi_{\rm B}(x_2)\phi_{\rm C}(x_3)\ldots|\Omega\rangle, \tag{191}$$

where Ω denotes the vacuum state, which is assumed to be unique up to a constant phase and to be invariant under transformation $U(a,\mathcal{A}) : U(a,\mathcal{A})|\Omega\rangle \equiv |\Omega\rangle$, where $U(a,\mathcal{A})$ is a continuous unitary representation of the inhomogeneous group SL(2,C): $\{a,\mathcal{A}\} \longrightarrow U(a,\mathcal{A})$, $a \in M_4$, $\mathcal{A} \in$ SL(2,C) associated with the restricted Lorentz group. The basic requirements for quantum fields $\{\Phi_{\Xi}(x)\}$, which afford possibilities for writing the Wightman functions for fields $\{\Phi_{\Xi}(x)\}$, besides those of rather strictly mathematical nature (i.e., concerning the state space $\mathcal{H}$ of the system considered, the domain of field operators, its density in $\mathcal{H}$, the hermiticity, the cyclicity of $|\Omega\rangle$, etc. [26] — [30]) are the following:

(A1) Relativistic invariance:

$$U(a,\mathcal{A})\phi_{\rm A}(x)U(a,\mathcal{A})^{-1} = \phi_{\rm A}(\mathcal{A}x + a), \tag{192}$$

for any $\rm A \in \Xi$,

(A2) The spectral condition: the eigenvalues of the energy momentum operator $\mathcal{P}_\mu$ lie in or on the plus (forward) light cone V_+.

The point $p = 0$ is the isolate eigenvalue of $\mathcal{P}$ to the eigenvector $|\Omega\rangle$ — the vacuum. The operator $\mathcal{P}^\mu\mathcal{P}_\mu \equiv m^2$ is interpreted as the square of the mass (the assumption (A2) is equivalent with requiring all energies to be positive). We have $U(a, \mathit{1}) \equiv \exp(i\mathcal{P}^\mu a_\mu)$.

(A3) Local commutativity (called also "microscopic causality"): for all pairs $\mathrm{A}, \mathrm{B} \in \Xi$

$$[\phi_{\mathrm{A}}(x),\ \phi_{\mathrm{B}}(y)] = 0, \tag{193}$$

if $(x - y)^2 < 0, x, y \in M_4$.

Having a set of fields satisfying the above assumptions and the vacuum state $|\Omega\rangle$, which is cyclic, one has a set of uniquely defined (by the relation (191)) Wightman functions $\{W^{(n)}\}$. Conversely, a knowledge of all Wightman functions $\{W^{(n)}\}$ is sufficient to characterize a quantum field theory completely. Any other quantum field theory with the same values for the $\{W^{(n)}\}$ is equivalent to the field theory from which the original $\{W^{(n)}\}$ were constructed, up to unitary transformation [26] — [31].

CPT–invariance could be shown to hold for any field theory which has properties listed above [26] — [29]. For example, the CPT–symmetry Θ has the property that, for charged scalar fields $\varphi_{\mathrm{A}}, \varphi_{\mathrm{B}}$

$$\Theta^{-1}\varphi_{\mathrm{A}}(x_1)\varphi_{\mathrm{B}}(x_2)\Theta = \varphi_{\mathrm{B}}(-x_2)^+ \ \ \varphi_{\mathrm{A}}(-x_1)^+, \tag{194}$$

which can be written for vacuum expectation values as follows

$$\langle\Omega|\varphi_{\mathrm{A}}(x_1)\varphi_{\mathrm{B}}(x_2)|\Omega\rangle = \langle\Omega|\varphi_{\mathrm{B}}(-x_2)^+\varphi_{\mathrm{B}}(-x_1)^+|\Omega\rangle. \tag{195}$$

For the considered case of neutral scalar fields, the requirement of CPT–invariance in terms of Wightman functions is written as follows

$$W^{(n)}_{\mathrm{AB...N}}(x_1, x_2, \ldots, x_n) = W^{(n)}_{\mathrm{N...B,A}}(-x_n, -x_{n-1}, \ldots, -x_1). \tag{196}$$

The essence of the CPT Theorem is contained in the following realization [26] — [29]:

CPT Theorem

If CPT–condition (196) holds for all $x_1, \ldots, x_n$ then for every $x_1, \ldots, x_n$ such that $x_1 - x_2, \ldots, x_{n-1} - x_n$ is a Jost point [26] — [31], the weak local commutativity condition

$$W^{(n)}_{\mathrm{AB...N}}(x_1, x_2, \ldots, x_n) = W^{(n)}_{\mathrm{N...B,A}}(x_n, x_{n-1}, \ldots, x_1) \tag{197}$$

holds. Conversely, if condition (197) holds in a (real) neighborhood of a Jost point, then CPT condition (196) holds everywhere.

Since (A3) implies the condition (197), every field theory of a local hermitian scalar field fulfilling (A1) and (A2) has CPT–symmetry.

Similar theorem is valid for other fields fulfilling the conditions listed above. One should notice that in the course of the proof of CPT Theorem, an essential role is played by the analytical properties of $\{W^{(n)}\}$ generated by (A2) [26] — [30].

Having a set of quantum fields, for which CPT Theorem is valid, one can construct CPT–invariant Lagrangian density, which leads to the CPT-invariant energy–momentum tensor and thus to the total energy of the system, i.e., to the CPT–invariant Hamiltonian H of this system.

All known CPT–invariance tests for neutral kaons are based on the LOY model, i.e., on the model containing, by assumption (106), exponentially decaying particles [1] — [7], [15] — [22]. The problem of the correctness of such models and their usefulness for designing these tests is explained by two following theorems (see [52]).

Theorem 2.

If there exists $|\psi\rangle \in \mathcal{H}$, $|\psi\rangle \neq 0$, $\langle\psi|\psi\rangle = 1$, where $\mathcal{H}$ is the total Hilbert state space of the system considered, such that

$$p(t;|\psi\rangle) = |a(t)|^2 \equiv |\langle\psi|\exp(-itH)|\psi\rangle|^2 \leq \exp(-\gamma|t|), \tag{198}$$

where $\gamma > 0$ and H is the total, selfadjoint Hamiltonian of the system under investigation, then the spectrum of H is the whole real line.

Proof: Using the projection valued measure $E(\lambda)$ for the Hamiltonian H: $H|\psi\rangle = \int \lambda \mathrm{d}E(\lambda)|\psi\rangle$, the amplitude $a(t)$ can be expressed as follows

$$a(t) = \int e^{-it\lambda}\mathrm{d} \parallel E(\lambda)|\psi\rangle\parallel^2. \tag{199}$$

From (198) it follows that

$$\sigma(\lambda) \stackrel{\text{def}}{=} \parallel E(\lambda)|\psi\rangle\parallel^2 \tag{200}$$

is analytically in the strip $|\mathrm{Im}\lambda| < \gamma/2$. For real λ, the function $\sigma(\lambda)$ defines the positive Stieltjes measure [53]. So, $a(t)$ is the Fourier transform of this measure $\sigma(\lambda)$. One can show that $\sigma(\lambda)$ is absolutely continuous and has a whole real line $\mathbf{R}^1$ as its support (for details see, e.g., [54]), which means that the spectrum of H is the whole real line [54]. (The proofs that the condition (198) implies unboundedness from below the spectrum of H also can be found in [55, 56, 57]).

Considering the case of the LOY model, one should rather use the next theorem, which is due to Williams [55]:

Theorem 3

Let $U_{\|}(t) = PU(t)P$, where $U(T) = \exp(-itH)$ is a strongly continuous, one parameter, unitary group on a Hilbert space $\mathcal{H} \supset \mathcal{H}_{\|} \equiv P\mathcal{H}$.

(i) If

$$\| U_{\|}(t)|\varphi\rangle \| \leq C \exp(-\gamma t) \tag{201}$$

for some $\gamma > 0$ and all $t > 0$, and for some nonvanishing $|\varphi\rangle$ in $\mathcal{H}_{\|}$, then the spectrum of H is the whole real line.

(ii) The same conclusion holds if we have exponential decrease only in the weak sense that there are two vectors $|\psi\rangle$ and $|\varphi\rangle$ in $\mathcal{H}_{\|}$ with $\langle\psi|\varphi\rangle \neq 0$ and

$$|\langle\psi|U_{\|}(t)|\varphi\rangle| \leq C \exp(-\gamma t) \tag{202}$$

for some $\gamma > 0$ and all $t > 0$.

The proof of this theorem can be found in [55].

Conclusions following from Theorems 2 and 3 are obvious: if the state space $\mathcal{H}$ of the system considered contains such the state $|\psi\rangle$ that the relation (198) holds, or if in the subspace unstable states $\mathcal{H}_{\|}$ of the system under consideration are vectors $|\mathbf{j}\rangle$, $(j = 1, 2)$, and $|\psi\rangle_{\|}$ such that $\langle\mathbf{j}|\psi\rangle_{\|} \neq 0$ and $|\langle\mathbf{j}|U_{\|}(t)|\psi\rangle_{\|}| \equiv |a_j(t)| \leq C \exp(-\gamma t)$ for some $\gamma > 0$ and all $t > 0$, ($a_j(t)$ is defined by (2)), then the CPT Theorem is not valid for these systems. This is because such the systems do not satisfy the assumption (A2), which is necessary for the proof of the CPT Theorem [26] — [31].

Summing up, the CPT Theorem cannot be proved for a system in which H has a spectrum equal to the whole real line, and therefore one finds that CPT–transformation cannot be considered as the symmetry in models containing an unstable state $|\psi\rangle$ (or unstable states $|\psi_\alpha\rangle$) with assumed exponential decay law of type (198) or (106). Simply, models with exponentially decaying particles can not be CPT–invariant! Therefore such the models cannot be used for studying CPT–invariance properties of physical systems and for designing CPT–invariance tests. For designing such the tests only such models can be used which are CPT–invariant or which are able to be CPT–invariant and CPT–noninvariant depending on parameters of these models. It seems to be logic that models only CPT–noninvariant can not be considered as appropriate for this purpose.

It seems that results of this Section explain the difference between predictions of the LOY model and conclusions described in the previous Section and following from the real properties of the system considered in the case nonconserved CPT–symmetry.

6 An alternative approximation for the effective Hamiltonian $H_{\|}$.

The aim of this Section is to show that there exist an approximation consistent with general properties following from the results of Sec. 4 for the effective Hamiltonian governing time evolution in a given subspace $\mathcal{H}_{\|}$ and to derive an approximate formulae for this effective Hamiltonians $H_{\|} \equiv H_{\|}(t)$, which CPT–transformation properties are consistent with those following from the real properties of the system under consideration , i.e., which diagonal elements, contrary to H_{LOY}, are not equal in CPT– invariant system (37). These approximate formulae for $H_{\|}(t)$ can be derived using the Krolikowski–Rzewuski equation for the projection of a state vector [58, 59] and [39] — [41], [51], [60, 61]. This equation results from the Schrödinger equation (2) for the total system under consideration [58, 59].

So, let us consider the evolution equations for orthogonal components $|\psi;t\rangle_{\|}$ (20) and for $|\psi;t\rangle_{\perp}$ (21) of the state vector $|\psi;t\rangle \equiv |\psi;t\rangle_{\|} + |\psi;t\rangle_{\perp}$ instead of the system equations for number functions $a_k(t)$, $F_J(\varepsilon;t)$. Using projection operators P and Q, (19), one can obtain from the Schrödinger equation (2) for the state vector $|\psi;t\rangle$ two equations for its orthogonal components $|\psi;t\rangle_{\|}$ and $|\psi;t\rangle_{\perp}$ valid for $t \geq t_0 = 0$:

$$
\begin{aligned}
i\frac{\partial}{\partial t}|\psi;t\rangle_{||} &= PHP|\psi;t\rangle_{||} + PHQ|\psi;t\rangle_{\perp}, && (203)\\
i\frac{\partial}{\partial t}|\psi;t\rangle_{\perp} &= QHQ|\psi;t\rangle_{\perp} + QHP|\psi;t\rangle_{||}, && (204)
\end{aligned}
$$

with the initial conditions (17), (18) and (105), which are equivalent to the following one

$$|\psi;t=0\rangle_{\perp} = 0. \tag{205}$$

Solving Eq (204) one can eliminate $|\psi;t\rangle_{\perp}$ from Eq (203) by substituting the solution of Eq (204) back into Eq (203). Looking for a solution of Eq (204) we can use the following substitution

$$|\widetilde{\psi;t}\rangle_{\perp} \stackrel{\text{def}}{=} e^{+itQHQ}|\psi;t\rangle_{\perp}. \;\; (t \geq 0), \tag{206}$$

By means of such a substitution Eq (204) can be replaced by the following one

$$
\begin{aligned}
i\frac{\partial}{\partial t}|\widetilde{\psi;t}\rangle_{\perp} &= e^{+itQHQ}QHP|\psi;t\rangle_{||}, \;\; (t \geq 0), && (207)\\
|\widetilde{\psi;t=0}\rangle_{\perp} &= 0.
\end{aligned}
$$

It is easy to solve this equation. Using its solution one finds

$$|\psi;t\rangle_{\perp} = -i\int_0^t e^{-i(t-\tau)QHQ}QHP|\psi;\tau\rangle_{||}\,d\tau, \;\; (t \geq 0). \tag{208}$$

Substituting (208) back into Eq (203) gives for $t \geq 0$:

$$i\frac{\partial}{\partial t}|\psi;t\rangle_{||} = PHP|\psi;t\rangle_{||} - i\int_0^t PHQe^{-i(t-\tau)QHQ}QHP|\psi;\tau\rangle_{||}\,d\tau. \tag{209}$$

Notice that Eq (209) is the exact one. (In the literature, equations of this type are called "master equation" [57, 62, 63], or Krolikowski–Rzewuski equation for the distinguished component of a state vector [40, 41, 51, 52, 60, 61]). It is convenient to rewrite this equation as follows

$$\begin{aligned} (i\frac{\partial}{\partial t} - PHP)|\psi;t\rangle_{||} &= -i\int_0^\infty K(t-\tau)|\psi;\tau\rangle_{||}d\tau, \\ |\psi;0\rangle_{||} &\neq 0, \quad |\psi;0\rangle_{\perp} = 0, \end{aligned} \tag{210}$$

where:

$$\begin{aligned} K(t) &= \Theta(t)PHQ\exp(-itQHQ)QHP, \\ Q &= \mathit{1} - P, \\ \Theta(t) &= \{1 \text{ for } t \geq 0, \ \ 0 \text{ for } t < 0\}. \end{aligned} \tag{211}$$

Using the property (171) one finds from (11), and (210)

$$V_{\|}(t)|\psi;t\rangle_{||} = -i\int_0^\infty K(t-\tau)|\psi;\tau\rangle_{||}d\tau \stackrel{\text{def}}{=} -iK * |\psi;t\rangle_{||}. \tag{212}$$

(Here the star $*$ denotes the convolution: $f*g(t) = \int_0^\infty f(t-\tau)g(\tau)\,d\tau$). Next, using this relation and a retarded Green's operator $G(t)$ for the equation (210)

$$G(t) = -i\Theta(t)\exp(-itPHP)P, \tag{213}$$

one obtains [41, 60, 61] for $|\psi;t\rangle_{||}$ having the form (22)

$$V_{\|}(t)\,U_{\|}(t)|\psi\rangle_{||} = -iK * \Big[\mathit{1} + \sum_{n=1}^{\infty}(-i)^n L * \ldots * L\Big] * U_{\|}^{(0)}(t)|\psi\rangle_{||}, \tag{214}$$

where L is convoluted n times, $\mathit{1} \equiv \mathit{1}(t) \equiv \delta(t)$,

$$L(t) = G * K(t), \tag{215}$$

and

$$U_{\|}^{(0)} = \exp(-itPHP)\ P \tag{216}$$

is a "free" solution of Eq.(210). Of course, the series (214) is convergent if $\| L(t) \| < 1$. If for every $t \geq 0$

$$\| L(t) \| \ll 1, \tag{217}$$

then, to the lowest order of $L(t)$, one finds from (214) [41, 60, 61]

$$V_{\|}(t) \cong V_{\|}^{(1)}(t) \stackrel{\text{def}}{=} -i \int_0^{\infty} K(t-\tau) \exp\left[i(t-\tau)PHP\right] d\tau. \tag{218}$$

This is a general formula, valid in any $\mathcal{H}_{||}$, for $V_{\|}(t) \cong V_{\|}^{(1)}(t)$ and thus (by (157) and (171)) for the approximate effective Hamiltonian $H_{||} = H_{||}(t) \cong H_{||}^{(1)}(t) \equiv PHP + V_{\|}^{(1)}(t)$.

To evaluate the integral (218) it is necessary to calculate $\exp[itPHP]$. Keeping in mind that in the case under studies PHP is the hermitian (2×2) matrix and using the Pauli matrices representation (see (24)),

$$PHP \equiv H_0 I_{\|} + \vec{H} \bullet \vec{\sigma} \equiv H_0\, P + \vec{H} \bullet \vec{\sigma}, \tag{219}$$

where $\vec{H}$ and $\vec{\sigma}$ denote the following vectors: $\vec{H} = (H_x, H_y, H_z)$, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, and the product $\vec{H} \bullet \vec{\sigma}$ is defined analogously to (25), one finds

$$e^{\pm itPHP} = e^{\pm itH_0}\Big[I_{\|}\, \cos(t\kappa) \pm i\frac{\vec{H} \bullet \vec{\sigma}}{\kappa} \sin(t\kappa)\Big], \tag{220}$$

where

$$H_0 = \frac{1}{2}[H_{11} + H_{22}],$$

$$\begin{aligned} (\kappa)^2 \stackrel{\text{def}}{=} \vec{H} \bullet \vec{H} &= (H_x)^2 + (H_y)^2 + (H_z)^2 \\ &\equiv H_{12}H_{21} + (H_z)^2, \end{aligned}$$

and

$$H_z = \frac{1}{2}[H_{11} - H_{22}].$$

It is convenient to use (219) again and replace $\vec{H} \bullet \vec{\sigma}$ by $\vec{H} \bullet \vec{\sigma} = PHP - H_0 P$ in Eq (220), which, after some algebra, gives

$$\begin{aligned} e^{+itPHP} \;\; &\equiv \;\; \frac{1}{2}e^{it(H_0+\kappa)}[(1 - \frac{H_0}{\kappa})P + \frac{1}{\kappa}PHP] \\ &+ \;\; \frac{1}{2}e^{it(H_0-\kappa)}[(1 + \frac{H_0}{\kappa})P - \frac{1}{\kappa}PHP]. \end{aligned} \tag{221}$$

This is a general form of the operator e^{itPHP}. Generally e^{itPHP} has the form (221) if PHP has nonzero off–diagonal matrix elements, ie., if the condition (176) holds.

The simplest case is the case, when

$$H_{12} = H_{21} = 0. \tag{222}$$

In this case one finds

$$PHP \equiv H_0\, P, \tag{223}$$

and then that

$$Pe^{itPHP} = Pe^{itH_0}, \tag{224}$$

and therefore the approximate formula (218) for $V_{\|}(t)$ yields

$$V_{\|}^{(1)}(t) = -PHQ \frac{e^{-it(QHQ - H_0)} - 1}{QHQ - H_0} QHP. \tag{225}$$

We are rather interested in the properties of the system at long time period, at the same for which the LOY approximation was calculated (see [Urb-Piskorski]), and therefore we will consider the properties of

$$V_{||} \stackrel{\text{def}}{=} \lim_{t \to \infty} V_{||}^{(1)}(t). \tag{226}$$

instead of the general case $V_{||}(t) \cong V_{||}^{(1)}(t)$.

At long time period, the relation (225) leads to

$$V_{||} = -\Sigma(H_0). \tag{227}$$

This means that in the case (223)

$$H_{||} = H_0\, P - \, \Sigma(H_0), \tag{228}$$

where the following definition was used,

$$H_{||} \stackrel{\text{def}}{=} \lim_{t \to \infty} H_{||}(t) \equiv PHP + \lim_{t \to \infty} V_{||}(t), \tag{229}$$

So, in the case of H such that the condition (222) occurs, one obtains the effective Hamiltonian $H_{||}$ which is exactly the same as in the LOY approach, $H_{||} \equiv H_{LOY}$, (compare (111)). This means that in such a case simply $(h_{11} - h_{22}) = 0$ when CPT symmetry is conserved.

In the general case (176) inserting (221) into (218) and then taking the limit (226) yields [36, 51]

$$\begin{aligned} V_{\|} \;\; = \;\; & -\frac{1}{2}\Sigma(H_0 + \kappa)\Big[(1 - \frac{H_0}{\kappa})P + \frac{1}{\kappa}PHP\Big] \\ & -\frac{1}{2}\Sigma(H_0 - \kappa)\Big[(1 + \frac{H_0}{\kappa})P - \frac{1}{\kappa}PHP\Big]. \end{aligned} \tag{230}$$

This approximate formula for $V_{\|}$ leads to the expressions for the matrix elements $v_{jk}(t)$, of $V_{||}$, and $v_{jk} = \lim_{t \to \infty} v_{jk}(t)$,

$$v_{jk} = \langle \mathbf{j} | V_{||} | \mathbf{k} \rangle, \quad (j, k = 1, 2), \tag{231}$$

and thus for $h_{jk}(t)$, and $h_{jk} = \lim_{t\rightarrow\infty} h_{jk}(t)$,

$$h_{jk} \equiv H_{jk} + v_{jk}, \qquad (j,k = 1,2), \tag{232}$$

having properties consistent with those following from the Schrödinger equation (2) and from general, universally valid, relations (157), (169) derived in Sec. 4. The difference between these formulae and those obtained within the LOY approach is especially visible if one considers the matrix elements $v_{jk}(t \rightarrow \infty) = v_{jk}$ of $V_{\parallel}(t \rightarrow \infty) = V_{\parallel} \cong V_{\parallel}^{(1)}(\infty)$ [60, 61], which can be used for practical calculations of CP– and CPT–violation parameters in neutral kaons complex. Without requiring conservation CP– or CPT–symmetry of the system considered, the following expressions for v_{jk} can be obtained [60]

$$\begin{aligned} v_{j1} = \; & - \; \frac{1}{2}\Big(1 + \frac{H_z}{\kappa}\Big)\Sigma_{j1}(H_0 + \kappa) - \frac{1}{2}\Big(1 - \frac{H_z}{\kappa}\Big)\Sigma_{j1}(H_0 - \kappa) \\ & - \; \frac{H_{21}}{2\kappa}\Sigma_{j2}(H_0 + \kappa) + \frac{H_{21}}{2\kappa}\Sigma_{j2}(H_0 - \kappa), \end{aligned} \tag{233}$$

$$\begin{aligned} v_{j2} = \; & - \; \frac{1}{2}\Big(1 - \frac{H_z}{\kappa}\Big)\Sigma_{j2}(H_0 + \kappa) - \frac{1}{2}\Big(1 + \frac{H_z}{\kappa}\Big)\Sigma_{j2}(H_0 - \kappa) \\ & - \; \frac{H_{12}}{2\kappa}\Sigma_{j1}(H_0 + \kappa) + \frac{H_{12}}{2\kappa}\Sigma_{j1}(H_0 - \kappa), \end{aligned} \tag{234}$$

where $j,k = 1,2$. Note that these formulae for v_{jk} and thus for h_{jk} have been derived without assuming any symmetries of a type CP–, T–, or CPT–symmetry for the total Hamiltonian H of the system considered. It should also be emphasized that all components of the expressions (233), (234) have the same order with respect to $\Sigma(\varepsilon)$.

If to assume that the condition (37) holds in the system under consideration, that is that CPT symmetry is conserved, then in the general case (176) one finds

$$H_{11} = H_{22} \equiv H_0, \tag{235}$$

which implies that

$$\kappa \;\equiv\; |H_{12}|, \tag{236}$$

$$H_z \;\equiv\; 0, \tag{237}$$

$$\Sigma_{11}(\varepsilon = \varepsilon^*) \;\equiv\; \Sigma_{22}(\varepsilon = \varepsilon^*) \stackrel{\text{def}}{=} \Sigma_0(\varepsilon = \varepsilon^*). \tag{238}$$

These relations caused by CPT symmetry of the system lead to the following expression for $V_{\parallel}^{\Theta} = lim_{t\rightarrow\infty} V_{\parallel}^{\Theta}(t)$, (where $V_{\parallel}^{\Theta}(t)$ denotes $V_{\parallel}(t)$ when (37) occurs),

$$\begin{aligned} V_{\parallel}^{\Theta} \;=\; & -\frac{1}{2}\Sigma(H_0 + |H_{12}|)\Big[\Big(1 - \frac{H_0}{|H_{12}|}\Big)P + \frac{1}{|H_{12}|}PHP\Big] \\ & -\frac{1}{2}\Sigma(H_0 - |H_{12}|)\Big[\Big(1 + \frac{H_0}{|H_{12}|}\Big)P - \frac{1}{|H_{12}|}PHP\Big]. \end{aligned} \tag{239}$$

The matrix elements v_{jk}^{Θ} of operator $V_{\|}^{\Theta}$ have the following form

$$\begin{aligned} v_{j1}^{\Theta} = & - \frac{1}{2}\Big\{\Sigma_{j1}(H_0+|H_{12}|)+\Sigma_{j1}(H_0-|H_{12}|) \\ & + \frac{H_{21}}{|H_{12}|}\Sigma_{j2}(H_0+|H_{12}|)-\frac{H_{21}}{|H_{12}|}\Sigma_{j2}(H_0-|H_{12}|)\Big\}, \end{aligned} \tag{240}$$

$$\begin{aligned} v_{j2}^{\Theta} = & - \frac{1}{2}\Big\{\Sigma_{j2}(H_0+|H_{12}|)+\Sigma_{j2}(H_0-|H_{12}|) \\ & + \frac{H_{12}}{|H_{12}|}\Sigma_{j1}(H_0+|H_{12}|)-\frac{H_{12}}{|H_{12}|}\Sigma_{j1}(H_0-|H_{12}|)\Big\}. \end{aligned} \tag{241}$$

The form of these matrix elements is rather inconvenient, e.g., for searching for their properties depending on the matrix elements H_{12} of PHP. It can be done relatively simply assuming [52, 61]

$$|H_{12}| \ll |H_0|. \tag{242}$$

Within such an assumption one finds [61, 52]

$$v_{j1}^{\Theta} \simeq -\Sigma_{j1}(H_0) - H_{21}\frac{\partial \Sigma_{j2}(x)}{\partial x}\Big|_{x=H_0}, \tag{243}$$

$$v_{j2}^{\Theta} \simeq -\Sigma_{j2}(H_0) - H_{12}\frac{\partial \Sigma_{j1}(x)}{\partial x}\Big|_{x=H_0}, \tag{244}$$

where $j = 1, 2$. One should stress that due to the presence of resonance terms, derivatives $\frac{\partial}{\partial x}\Sigma_{jk}(x)$ need not be small, and so the products $H_{jk}\frac{\partial}{\partial x}\Sigma_{jk}(x)$ in (243), (244).

From this formulae we conclude that in CPT invariant system, e.g., the difference between the diagonal matrix elements which plays an important role in designing tests of type (96) for the neutral kaons system, equals to the lowest order of $|H_{12}|$ [64],

$$h_{11}^{\Theta} - h_{22}^{\Theta} \simeq H_{12}\frac{\partial \Sigma_{21}(x)}{\partial x}\Big|_{x=H_0} - H_{21}\frac{\partial \Sigma_{12}(x)}{\partial x}\Big|_{x=H_0} \equiv 2h_z^{\Theta} \neq 0. \tag{245}$$

So, in a general case, in contradiction to the property (131) obtained within the LOY theory, one finds for diagonal matrix elements of $H_{\|}$ calculated within the above described approximation that in CPT–invariant system the nonzero matrix elements, $H_{12} \neq 0$, of PHP cause that

$$(h_{11}^{\Theta} - h_{22}^{\Theta}) \neq 0. \tag{246}$$

(If CPT symmetry holds , $2h_z^{\Theta} = 0$ only if $[\mathcal{CP}, H] = 0$).

Comparing relations (240), (241) and (109) one can relate matrix elements, h_{jk}^{Θ} of the more accurate effective Hamiltonian, $H_{||}$, considered in this Section, to the corresponding matrix elements h_{jk}^{LOY} of H_{LOY}. So, assuming that (242)) holds one finds in the CPT–invariant system

$$h_{j1}^{\Theta} \simeq h_{j1}^{\mathrm{LOY}} - H_{21}\frac{\partial \Sigma_{j2}(x)}{\partial x}\Big|_{x=H_0}, \tag{247}$$

$$h_{j2}^{\Theta} \simeq h_{j2}^{\mathrm{LOY}} - H_{12}\frac{\partial \Sigma_{j1}(x)}{\partial x}\Big|_{x=H_0}, \tag{248}$$

where $j = 1, 2$.

Eigenvectors, $|l(s)\rangle$, of this $H_{||}^{\Theta}$ for the eigenvalues $\mu_{l(s)}$, (see (62) and (59)) differ from the corresponding eigenvectors of H_{LOY}^{Θ}, (see (133) — (138)). In the case considered one finds (see (61))

$$\alpha_l + \alpha_s = \frac{h_{11}^{\Theta} - h_{22}^{\Theta}}{h_{12}^{\Theta}} \neq 0, \tag{249}$$

and

$$|\alpha_l| \neq |\alpha_s|, \tag{250}$$

whereas for H_{LOY}^{Θ} one has (see (135))

$$\alpha_l^{LOY} + \alpha_s^{LOY} \equiv 0, \quad \text{and} \quad |\alpha_l^{LOY}| = |\alpha_s^{LOY}|.$$

Analogously with (247) and (248), to the lowest order of $|H_{12}|$, for eigenvalues μ_l, μ_s (62) of $H_{||}$, we obtain [61]

$$\begin{aligned}
\mu_s^{\Theta} \simeq \mu_s^{LOY} - \frac{1}{2} \Big[& H_{12}\Big(\frac{\partial \Sigma_{21}(x)}{\partial x}\Big|_{x=H_0} + a\frac{\partial \Sigma_0(x)}{\partial x}\Big|_{x=H_0}\Big) \\
+ & H_{21}\Big(\frac{\partial \Sigma_{12}(x)}{\partial x}\Big|_{x=H_0} + a^{-1}\frac{\partial \Sigma_0(x)}{\partial x}\Big|_{x=H_0}\Big)\Big], \\
\mu_l^{\Theta} \simeq \mu_l^{LOY} - \frac{1}{2} \Big[& H_{12}\Big(\frac{\partial \Sigma_{21}(x)}{\partial x}\Big|_{x=H_0} - a\frac{\partial \Sigma_0(x)}{\partial x}\Big|_{x=H_0}\Big) \\
+ & H_{21}\Big(\frac{\partial \Sigma_{12}(x)}{\partial x}\Big|_{x=H_0} - a^{-1}\frac{\partial \Sigma_0(x)}{\partial x}\Big|_{x=H_0}\Big)\Big],
\end{aligned} \tag{251}$$

where a is defined as follows

$$a \stackrel{\text{def}}{=} \Big(\frac{h_{21}^{LOY}}{h_{12}^{LOY}}\Big)^{1/2}, \tag{252}$$

and μ_s^{LOY}, μ_l^{LOY} are eigenvalues of H_{LOY} for eigenstates $|K_s>$ and $|K_l>$ respectively (see (138)).

Note that the relation (245) means means that if CPT–symmetry is conserved in the system considered and (242) holds then for the parameter δ (75) one finds that

$$\delta^{\Theta} \equiv \frac{2h_z^{\Theta}}{D^{\Theta}} \neq 0, \tag{253}$$

where

$$D^{\Theta} \simeq D^{LOY} - (H_{12} + H_{21}a^{-1})(1+a)\frac{\partial \Sigma_0(x)}{\partial x}\Big|_{x=H_0},$$

and $D^{LOY} = h_{12}^{LOY} + h_{21}^{LOY} + \Delta\mu^{LOY}$, $\Delta\mu^{LOY} \stackrel{\text{def}}{=} \mu_s^{LOY} - \mu_l^{LOY}$.

Many test of fundamental symmetries in neutral kaon and similar complexes make use of relations (91) — (97) and the results of such tests are interpreted within properties of matrix elements of the LOY effective Hamiltonian. Analysis of properties of matrix elements of the exact effective Hamiltonian performed in Sec. 4 and relations (245) — (252) shows that such an interpretation need not be correct and it need not reflect the real properties of the Nature. As an example let us analyze the test based on the relation (96). The standard interpretation of such a test as the CPT invariance test follows from properties (131) and (143) of the LOY approximation. So, let us consider in details the relation (96) using matrix elements h_{jk} of the more accurate effective Hamiltonian. From the formula (245) it follows that the left side of the relation (96) takes the following form in the case of very weak interactions allowing for the nonzero first order transitions $|\mathbf{1}> \rightleftharpoons |\mathbf{2}>$, that is in the case when $H_{12} \neq 0$ and the property (242) holds,

$$M_1 - M_2 \; = \; \Re\,(h_{11}^{\Theta} - h_{22}^{\Theta}) = 2\,\Im\,\Big(H_{21}\frac{\partial \Sigma_{12}^{I}(x)}{\partial x}\Big|_{x=H_0}\Big) \; + \; \ldots \; \neq \; 0. \tag{254}$$

(Note that as a matter of fact assuming (176) one has $H_{21} \equiv <\mathbf{2}|H^{(1)}|\mathbf{1}>$ in (254)). Thus taking into account this result and the implications of the assumptions (223), (222) one can conclude that [64]

$$\Re\,(h_{11}^{\Theta} - h_{22}^{\Theta}) \; = 0 \; \Leftrightarrow \; |H_{12}| \; = \; 0 \tag{255}$$

within the considered approximation. Finally, using result (254) one can replace the relation (96) by the following one:

$$\begin{aligned} 2\,\Im\Big(<\mathbf{2}|H^{I}|\mathbf{1}> \frac{\partial \Sigma_{12}^{I}(x)}{\partial x}\Big|_{x=H_0}\Big) \; &\simeq \; -(\gamma_s - \gamma_l)[\tan\phi_{SW}\,\Re\,(\frac{\varepsilon_s - \varepsilon_l}{2}) \\ &\quad -\Im\,(\frac{\varepsilon_s - \varepsilon_l}{2})\,], \qquad (256) \\ &\equiv \; -(\gamma_s - \gamma_l)[\tan\phi_{SW}\,\Re\,(\delta) \\ &\quad -\Im\,(\delta)\,], \qquad (257) \end{aligned}$$

Note that this relation was derived within the more accurate approximation discussed in this Section assuming CPT invariance of the total system under consideration.

At the end of this Section it should be pointed out that the more accurate than the LOY approximation (218) considered in this Section and leading to the formulae (233), (234) for matrix elements of the more accurate effective Hamiltonian, $H_{||}$, is self-consistent and well defined.

7 Final remarks

Discussing properties of the LOY model, it should be noticed that the assumption (106) does not only change the integration properties of Eq. (102) essentially making the integration of this equation easier, but also it essentially changes the analytical properties of the amplitude $F_j(\omega, t)$ and thus amplitudes $a_k(t)$ leading to definite properties of matrix elements of H_{LOY}. According to the Theorem 3 of Sec. 5, these changes of analytical properties of amplitudes considered cause the LOY model to behave like a system with the Hamiltonian unbounded from below. Therefore CPT transformation cannot be the symmetry for such an obtained model, even if it was the symmetry for the initial system. This means that solutions of Eq. (2), or Eqs. (101), (102) and solutions of Eq. (108) should have different CPT–transformation properties. (This reservation need not take place in the case of the CP– transformation).

On the other hand, from basic principles of Quantum Theory, it follows that exponential decay fails at short and at long time regions [65, 66]. It seems that this and results of Sec. 5 mentioned explain why the assumption (106) on exponentiality of decay amplitudes of K_0, $\overline{K}_o$ mesons is wrong if one consider such tiny effects as possible violations of CPT–symmetry. In the light of the conclusions following from Theorems 2 and 3 of Sec. 5 and from above remarks, it should be clear that relation (131) $h_{11}^{LOY} = h_{22}^{LOY}$ need not be true for real CPT–invariant systems. What is more, in view of the conclusions obtained in Sec. 4, the conventional, standard interpretation of the difference of these matrix elements appears to be wrong.

Similar reservations concern formulae for the matrix elements of H_{eff} obtained within the so–called pole approximation [4], which is, in fact, equivalent to the LOY approach. This approximation is obtained by replacing the function (the matrix $\Sigma(\varepsilon)$) which appears in the denominator under the integral defining the amplitudes for which we are searching, and which corresponds to a real system, by its value at the pole m_0 (i.e., one replaces $\Sigma(\varepsilon)$ by $\Sigma(m_0)$ there). This substitution completely changes the analytical properties of this function and therefore the approximate model obtained is no longer able to describe all, perhaps, very tiny effects (such as possible violations of CPT–symmetry) occurring in the real system. The same concerns models of neutral kaons decay in which Bell–Steinberger unitarity relation is assumed to be valid: the exponential decay laws for states $|K_0\rangle$ and $|\overline{K}_0\rangle$ was

assumed in the course of the derivation of this relation [67].

Taking into account conclusions of Sec. 4, the experimental result $\delta \propto (h_{11} - h_{22}) \neq 0$ means nothing: in such a case the relations $[\Theta, H] = 0$ and $[\Theta, H] \neq 0$ are admissible in the system under consideration. It is clear that it will be essential for the result of experimental tests of the CPT–invariance $\delta = 0$ to be exact and only such a result can be understood independently of the model. In contradistinction to the standard, conventional interpretation [2] — [8], [15] — [22] such a result will prove that $[\Theta, H] \neq 0$ in neutral kaons, or other similar, systems. The problem is whether the experimenter will be able to perform their experiments with the accuracy guaranteeing the proper answer to the question of whether $\delta = 0$ or $\delta \neq 0$. There is a chance for the tested system that $[\Theta, H] = 0$ only if the experiment confirms the existence of a positive small (maybe very, very small) number, say λ, such that $|\delta| > \lambda$. The proper interpretation of the result $\delta \neq 0$ depends on the model calculations of the quantity $(h_{11}(t) - h_{22}(t))$, or, which is equivalent, on the calculated values of matrix elements of type $\langle \mathbf{2}|\mathcal{A}(t)|\mathbf{1}\rangle$ and $\langle \mathbf{2}|\mathcal{B}(t)|\mathbf{1}\rangle$. This can not be performed within the LOY approach and requires more exact approximations. It seems that the approximation described in [10] or, the other one, described in Sec.6 and exploited in [60, 61, 52] may be a more effective tool for this purpose: assuming (37), (38), $(h_{11} - h_{22}) \neq 0$ was found in [52, 60, 61] within those approximations.

Tests consisting of a comparison of the equality of the decay laws of K_0 and $\overline{\mathrm{K}}_0$ mesons, i.e. verifying the relation (155), seem to be the only completely model independent tests for verifying the CPT–invariance in such and similar systems.

Taking into account all the above, it seems that all theories describing the time evolution of the neutral kaons and similar systems by means of the effective Hamiltonian $H_{\|}$ governing their time evolution in which the CPT–invariance of $H_{\|}$ as a symmetry generated by CPT—symmetry of the total Hamiltonian H and leading to the property (109), is assumed, are unable to give the exact and correct description of all aspects of the effects connected with the violation or non– violation of the CP– and, especially, CPT–symmetries. Also, it seems that results of the experiments with neutral kaons, etc., designed and carried out on the basis of expectations of theories within LOY approximation, such as tests of CPT–invariance (at least results of those in which CPT–invariance or CPT–noninvariance of $H_{\|}$ generated by such the invariance properties of H were essential), should be revised using other methods than the LOY approach (e.g. using a formalism based on the Fock-Krylov theorem [68] and exploited in [65, 66], or the approach proposed in Sec. 6).

Using the formalism briefly described in previous Section, one can find $(h_{11} - h_{22})$ for the generalized Fridrichs–Lee model [11, 12, 60]. Within this toy model one finds [52], [64]

$$\begin{aligned} \Re\,(h_{11} - h_{22}) \stackrel{\text{df}}{=} \Re\,(h_{11}^{FL} - h_{22}^{FL}) &\simeq i\frac{m_{21}\Gamma_{12} - m_{12}\Gamma_{21}}{4(m_0 - \mu)} \\ &\equiv \frac{\Im\,(m_{12}\Gamma_{21})}{2(m_0 - \mu)}. \end{aligned} \tag{258}$$

This estimation has been obtained in the case of conserved CPT–symmetry for $|m_{12}| \ll (m_0 - \mu)$, which corresponds to (242). In (258) Γ_{12}, Γ_{21} can be identified with those appearing in the LOY theory, $m_0 \equiv H_{11} = H_{22}$ can be considered as the kaon mass [11], $m_{jk} \equiv H_{jk}\,(j,k = 1,2)$, μ can be treated as the mass of the decay products of the neutral kaon [11].

For the neutral K-system, to evaluate $(h_{11}^{FL} - h_{22}^{FL})$ one can follow, e.g., [7, 11] and one can take $\frac{1}{2}\Gamma_{21} = \frac{1}{2}\Gamma_{12}^{*} \sim \frac{1}{2}\Gamma_s \sim 5 \times 10^{10}\text{sec}^{-1}$ and $(m_0 - \mu) = m_K - 2m_\pi \sim 200$ MeV $\sim 3 \times 10^{23}\text{sec}^{-1}$ [18]. Thus

$$\Re\,(h_{11} - h_{22}) \sim \frac{\Gamma_s}{4(m_K - 2m_\pi)}\,\Im\,(H_{12}), \tag{259}$$

that is,

$$|\Re\,(h_{11}^{FL} - h_{22}^{FL})| \sim 1,7 \times 10^{-13} |\Im\,(m_{12})| \equiv 1,7 \times 10^{-13} |\Im\,(H_{12})|. \tag{260}$$

Note that the relation (259) is equivalent to the following one

$$\Re\,(h_{11} - h_{22}) \sim -i\frac{\Gamma_s}{4(m_K - 2m_\pi)} < \mathbf{1}|H_-|\mathbf{2} >, \tag{261}$$

where H_- is the CP odd part of the total Hamiltonian $H \equiv H_+ + H_-$. There are $H_- \stackrel{\text{def}}{=} \frac{1}{2}[H - (\mathcal{CP})H(\mathcal{CP})^+]$ and $H_+ \stackrel{\text{def}}{=} \frac{1}{2}[H + (\mathcal{CP})H(\mathcal{CP})^+]$ (see [19, 22]). H_+ denotes the CP even part of H. We have $< \mathbf{1}|H_-|\mathbf{2} > \equiv i\Im\,(< \mathbf{1}|H|\mathbf{2} >)$ $= i\,\Im\,(H_{12})$. According to the literature, in the case of the superweak model for CP violation it should be $< \mathbf{1}|H_-|\mathbf{2} > \equiv i\Im\,(H_{12}) \neq 0$ to the lowest order and $< \mathbf{1}|H_-|\mathbf{2} >= 0$ in the case of a miliweak model [19, 22].

For the Fridrichs–Lee model it has been found in [60] that $h_{jk}(t) \simeq h_{jk}$ practically for $t \geq T_{as} \simeq \frac{10^2}{\pi(m_0 - |m_{12}| - \mu)}$. This leads to the following estimation of T_{as} for the neutral K–system: $T_{as} \sim 10^{-22}$ sec.

Dividing both sides of (260) by m_0 one arrives at the relation corresponding to (145):

$$\frac{|\Re\,(h_{11}^{FL} - h_{22}^{FL})|}{m_0} \sim 1,7 \times 10^{-13} \frac{|\Im\,(m_{12})|}{m_0} \equiv 1,7 \times 10^{-13} \frac{|\Im\,(H_{12})|}{m_0}. \tag{262}$$

So, if we suppose for a moment that the result (145) is the only experimental result for neutral K complex then it is sufficient for $\frac{|\Im\,(H_{12})|}{m_0}$ to be $\frac{|\Im\,(H_{12})|}{m_0} < 10^{-5}$ in order to fulfill the estimation (145). Of course this could be considered as the upper bound for a possible value of the ratio $\frac{|\Im\,(H_{12})|}{m_0}$ only if there were no other experiments and no other data for the $K_0, \overline{K}_0$ complex. Note that form such a point o view the suitable order of $\frac{|\Im\,(H_{12})|}{m_0}$ is easily reached by the hypothetical Wolfenstein superweak interactions [17, 69], which admits first order $|\Delta S| = 2$ transitions $K_0 \rightleftharpoons \overline{K}_0$, that is, which assumes a non–vanishing first order transition

matrix $H_{12} =< \mathbf{1}|H^I|\mathbf{2} >\sim gG_F \neq 0$ with $g \ll G_F$. The more realistic estimation for $\frac{|\Im\,(H_{12})|}{m_0}$ can be found using the property $\frac{|\Im\,(H_{12})|}{m_0} \equiv \frac{|<\mathbf{1}|H_{-}|\mathbf{2}>|}{m_0}$. One can assume that $\frac{|<\mathbf{1}|H_{-}|\mathbf{2}>|}{m_0} \sim \frac{H_{-}}{H_{strong}}$. There is $\frac{H_{-}}{H_{strong}} \sim 10^{-14}|\varepsilon|$ for the case of the hypothetical superweak interactions (see [19], formula (15.138)) and thus $\frac{|\Im\,(H_{12})|}{m_0} \sim 10^{-14}|\varepsilon|$. (Using this last estimation one should remember that it follows from the LOY theory of neutral K complex). This estimation allows one to conclude that

$$\frac{|\Re\,(h_{11}^{FL} - h_{22}^{FL})|}{m_0} \sim 1,7 \times 10^{-27}|\varepsilon|. \tag{263}$$

This estimation is the estimation of type (145) and it can be considered as a lower bound for $\frac{|\Re\,(h_{11} - h_{22})|}{m_0}$. (see also [70]).

Note that contrary to the approximation described in Sec. 6, the LOY approximation, as well as the similar approximation leading to the Bell–Strinberger unitary relations [67] are unable to detect and correctly identify effects caused by the existence (or absence) of the interactions for which $H_{12} \neq 0$ in the system.

Let us analyze some important observations following from (254), (256) and from the rigorous result obtained in [42]. The non–vanishing of the right hand side of the relation (96) can not be considered as the proof that the CPT–symmetry is violated. So, there are two general conclusions following from (254), (255), (256) and [42, 46]. The first one: the tests based on the relation (96) can not be considered as CPT–symmetry tests and this is the main conclusion of this paper. The second one: such tests should rather be considered as the tests for the existence of new hypothetical (superweak (?)) interactions allowing for the first order $|\Delta S| = 2$ transitions. Simply, the left hand side of the relation (254) can differ from zero only if the matrix element $< \mathbf{2}|H|\mathbf{1} >$ is different from zero and thus the nonzero value of the right hand side of the relation (256) means that it should be $< \mathbf{2}|H^I|\mathbf{1} >\neq 0$.

Note that within the LOY theory one can also obtain nonzero first order $|\Delta S| = 2$ transitions in Standard Model for $K_0 - \overline{K}_0$ complex [20]. The main difference between such an effect and the effect discussed in this paper and connected with the relations (96), (254) — (256) is that within the LOY theory the first order $|\Delta S| = 2$ transitions can appear only for off–diagonal matrix elements, h_{jk}^{LOY}, $(j \neq k)$, of the effective Hamiltonian, H_{LOY}, whereas within the more accurate approximation, discussed in the previous Section, diagonal matrix elements, h_{11}, h_{22}, as well as off–diagonal matrix elements of the effective Hamiltonian $H_{||}$ depend on H_{12}, H_{21}. Within the LOY approach, diagonal matrix elements of H_{LOY} do not depend on H_{12}, H_{21}. Therefore the effect discussed in this paper is absent in the LOY theory.

On the other hand, one should remember that the non–vanishing right hand side of the relations (96), (256) can be considered as the conclusive proof that new interactions allowing for the first order $|\Delta S| = 2$ transitions $K_0 \rightleftharpoons \overline{K}_0$ exist only if an another experiment, based on other principles, definitively confirms that the CPT–symmetry is not violated in $K_0 - \overline{K}_0$ system.

Unfortunately the accuracy of the today's experiments is not sufficient to improve the estimation (145) to the order required by (262). This especially concerns the accuracy required by our "more realistic estimation" for $\frac{|\Im(H_{12})|}{m_0}$. Simply it is beyond today's experiments reach. In the light of the above estimations, keeping in mind (254), only much more accurate tests based on the relation (96) can give the answer whether the mentioned new hypothetical interactions exists or not.

It also seems, that above results have some meaning when attempts to describe possible deviations from conventional quantum mechanics are made and when possible experimental tests of such a phenomenon and CPT–invariance in the neutral kaons system are considered [71, 72]. In such a case a very important role is played by nonzero contributions to $(h_{11} - h_{22})$ [71, 72]: The correct description of these deviations and experiments mentioned is impossible without taking into account the results of this Section and the above Sections 4 — 6. This can not be performed within the LOY approach and requires more exact approximations. It seems that the approximation described and exploited in [41, 60, 61] may be a more effective tool for this purpose.

Last remark, other results [60, 61, 52] obtained within the approximation described in Sec. 6 suggest also that the form of other parameters usually used to describe properties of $K_0 - \overline{K}_0$ system is different for the case $H_{12} \neq 0$ and for the case $H_{12} = 0$. This can be used as the basis for designing other tests for the hypothetical new interactions.

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